Foundations for Abstract Semantics

Roger Bishop Jones

22 February 2005

Contents

1 Introduction **1**
- 1.1 Descriptive Language 2
- 1.2 Entailment and Analyticity 2
- 1.3 Abstract Semantics 2

2 Foundational Problems **2**
- 2.1 Platonism and Objectivity 2
- 2.2 The Role of Foundations 2

3 The Semantics of Set Theory **2**

1 Introduction

The ultimate concern of this essay is with the question of *whether* and *how* it may be possible to *define* the semantics of languages.

First let us briefly consider *why* definitions of semantics might be desirable.

Science, and the engineering and technology which it supports and which transform our prosperity and way of life, can be idealised as approximating to a *deductive-nomological* method. In this, both the application and the verification of scientific theories depend upon the use of deduction for deriving from scientific theories particular predictions about how the world will behave. There are of course, many further applications for deductive inference outside of science.

Here we are concerned with aspects of the underpinnings of deduction. In historically recent times, the analysis of deduction has lead to its almost complete formalization. The availability of formal tests which are sufficient to establish that an inference is deductively sound.

One of the problems which arise in considering *whether* such definitions are possible is *the problem of regress*. If a definition is to be given, it must surely be given in a language whose semantics is itself well-defined. This suggests that we may need to distinguish between the *object language* whose semantics we are defining and the *meta language* in which the definition is to be expressed, and that an infinite sequence of *meta languages* might arise. The concern with issues such as this is what makes the discussion *foundational*.

It is my consideration of only some aspects of semantics which leads me to speak of *abstract semantics*. An *abstract semantics* is one which makes reference only to *abstract entities* not to those concrete entities such as tables and chairs which might distinguish the world we live in from other possible worlds. My reason for considering such semantics as can be given in terms of abstract entities is that methods sufficient for this kind of semantics are quite different to those needed for concrete semantics, but that, for the purposes of deductive reasoning, abstract semantics suffices.

Knowledge derived from or supported by deductive proof is often supposed to be among the most reliable knowledge that we have, paradigm cases including the theorems of mathematics. Such knowledge is nevertheless subject to skeptical arguments, and sometimes propositions thought to have been established by these means are found to be false. Even deductive methods fail to confer absolute certainty.

This essay is concerned with a kind of response to skeptical arguments which bear upon deductive methods.

Insofar as skeptical arguments speak against the possibility of absolutely certain knowledge the response given here is one of aquiescence. This aquiescence extends also to skepticism about absolutely certain knowledge of unconditional probabilities.

Many people nevertheless have, and are justified in having, a high degree of confidence in the reliability of deductive methods. It is nevertheless the case that some ways with deduction are more reliable than others, and that certain kinds of foundational (and other) studies can be instrumental in establishing deductive methods which are more reliable than others. Deductive methods have been effective since antiquity, but theoretical advances in mathematical logic and technological advances in electronic and information engineering have made possible methods of establishing deductive conclusions which are substantially more reliable than hitherto, and which confer higher degrees of confidence and assurance in the truth of their conclusions. These facts speak against the attachment of too great a significance to skeptical arguments.

1.1 Descriptive Language

1.2 Entailment and Analyticity

1.3 Abstract Semantics

2 Foundational Problems

2.1 Platonism and Objectivity

2.2 The Role of Foundations

3 The Semantics of Set Theory