A Foundational Programme

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Contents

Preface

Early versions of this document are primarily intended to give a set of references or links to the fragmentary documentation of my foundational thinking. Later perhaps a fuller exposition of where I think I am heading.

Chapter 1

Introduction

1.1 First Ideas

My interest in the foundations of mathematics dates back to at least 1971. My first acquaintance with the topic probably was Russell's *Introduction to Mathematical Philosophy*[Rus19], unless one counts what Ayer had to say on the topic in *Language Truth and Logic*[Aye71]. From these two sources I became acquainted with two ideas which have remained important to me since.

First is the logicist thesis that *mathematics is logic*. Though Russell's exposition on this is much more detailed, Ayer's presentation of it as the observation that mathematics is *analytic* and that its status as logic (in what must now be called 'the broad sense') is independent of whatever arrangements we may chose for demonstrating mathematical propositions, has been the sense in which I have remained a logicist (without a hint of perturbation or doubt) ever since. It was not until much later that I became well enough acquainted with the work of Rudoolf Carnap, to understand him as a principalk source of my own foundational inclinations.

Second is the idea of a *formal logical foundation system for mathematics*. This idea is found in Frege and in Russell. It is most clearly articulated by Frege's dictum that:

$$\text{Mathematics} = \text{Logic} + \text{Definitions}$$

In which the 'logic' is what I have called above a *logical foundation system*. It is unimportant (for me) whether we call this central core 'logic' or 'set theory' or something else altogether. The important thing

1.1.1 Combinatory Logic

Persistent Applicative Heaps and Knowledge Bases[Jond]
Logical foundations and formal verification[Jonb]
Creative foundations and for program verification[Jona]
Logical Necessity and the Foundations of Mathematics[Jonc]

1.2 Alternate Ontologies

1.2.1 Well-Founded

1.2.2 Non-Well-Founded

1.2.3 Polysets

The Theory of PolySets and its Applications[Jone]
PolySets: foundational ontologies for formal mathematics (presentation notes)[Jon08a][Jon08b]
PolySet Theory[**?**]

1.2.4 Infinitary Comprehension

Set Theory as Consistent Infinitary Comprehension[Jon10d]
Infinitarily Definable Non-Well-Founded Sets[Jon10a]
Infinitarily Definable Sets[Jon10b]
Infinitary First Order Set Theory[Jon10c]

1.2.5 Infinitary Combinators and Finitary Illative Lambda-Calculus

An Illative Lambda-Calculus[Jon0]

1.3 Foundational Architecture

Bibliography

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