# Analyses of Analysis: Volume III - Theory Listings

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# Chapter 1

# Introduction

## Chapter 2

# Plato and Aristotle

## 2.1 The Theory syllog1

| | *aristotle* |
|---|---|
| ***TermS*** | *TermS* |
| ***Barbara*** | $M \subseteq P, \mathbb{S} \subseteq M \vdash \mathbb{S} \subseteq P$ |
| ***Celarent*** | $M \subseteq \sim P, \mathbb{S} \subseteq M \vdash \mathbb{S} \subseteq \sim P$ |
| ***Darii*** | $M \subseteq P, \neg\, \mathbb{S} \subseteq \sim M \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Ferio*** | $M \subseteq \sim P, \neg\, \mathbb{S} \subseteq \sim M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Cesare*** | $P \subseteq \sim M, \mathbb{S} \subseteq M \vdash \mathbb{S} \subseteq \sim P$ |
| ***Camestres*** | $P \subseteq M, \mathbb{S} \subseteq \sim M \vdash \mathbb{S} \subseteq \sim P$ |
| ***Festino*** | $P \subseteq \sim M, \neg\, \mathbb{S} \subseteq \sim M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Baroco*** | $P \subseteq M, \neg\, \mathbb{S} \subseteq M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Disamis*** | $\neg\, M \subseteq \sim P, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Datisi*** | $M \subseteq P, \neg\, M \subseteq \sim \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Bocardo*** | $\neg\, M \subseteq P, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Ferison*** | $M \subseteq \sim P, \neg\, M \subseteq \sim \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Camenes*** | $P \subseteq M, M \subseteq \sim \mathbb{S} \vdash \mathbb{S} \subseteq \sim P$ |
| ***Dimaris*** | $\neg\, P \subseteq \sim M, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Fresison*** | $P \subseteq \sim M, \neg\, M \subseteq \sim \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Darapti*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, M \subseteq P, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Felapton*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, M \subseteq \sim P, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Bramantip*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, P \subseteq M, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Fesapo*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, P \subseteq \sim M, M \subseteq \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Barbari*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, M \subseteq P, \mathbb{S} \subseteq M \vdash \neg\, \mathbb{S} \subseteq \sim P$ |
| ***Celaront*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, M \subseteq \sim P, \mathbb{S} \subseteq M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Cesaro*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, P \subseteq \sim M, \mathbb{S} \subseteq M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Camestrop*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, P \subseteq M, \mathbb{S} \subseteq \sim M \vdash \neg\, \mathbb{S} \subseteq P$ |
| ***Camenop*** | $\neg\, \{\} \in \{M; \mathbb{S}; P\}, P \subseteq M, M \subseteq \sim \mathbb{S} \vdash \neg\, \mathbb{S} \subseteq P$ |

## 2.2 The Theory syllog2

| | |
|---|---|
| | *aristotle* |
| ***TermP*** | *TermP* |
| ***Barbara*** | $M \Rightarrow P,\ \mathbb{S} \Rightarrow M \vdash \mathbb{S} \Rightarrow P$ |
| ***Celarent*** | $M \Rightarrow \neg\ P,\ \mathbb{S} \Rightarrow M \vdash \mathbb{S} \Rightarrow \neg\ P$ |
| ***Darii*** | $M \Rightarrow P,\ \neg\ (\mathbb{S} \Rightarrow \neg\ M) \vdash \neg\ (\mathbb{S} \Rightarrow \neg\ P)$ |
| ***Ferio*** | $M \Rightarrow \neg\ P,\ \neg\ (\mathbb{S} \Rightarrow \neg\ M) \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |
| ***Cesare*** | $P \Rightarrow \neg\ M,\ \mathbb{S} \Rightarrow M \vdash \mathbb{S} \Rightarrow \neg\ P$ |
| ***Camestres*** | $P \Rightarrow M,\ \mathbb{S} \Rightarrow \neg\ M \vdash \mathbb{S} \Rightarrow \neg\ P$ |
| ***Festino*** | $P \Rightarrow \neg\ M,\ \neg\ (\mathbb{S} \Rightarrow \neg\ M) \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |
| ***Baroco*** | $P \Rightarrow M,\ \neg\ (\mathbb{S} \Rightarrow M) \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |
| ***Disamis*** | $\neg\ (M \Rightarrow \neg\ P),\ M \Rightarrow \mathbb{S} \vdash \neg\ (\mathbb{S} \Rightarrow \neg\ P)$ |
| ***Datisi*** | $M \Rightarrow P,\ \neg\ (M \Rightarrow \neg\ \mathbb{S}) \vdash \neg\ (\mathbb{S} \Rightarrow \neg\ P)$ |
| ***Bocardo*** | $\neg\ (M \Rightarrow P),\ M \Rightarrow \mathbb{S} \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |
| ***Ferison*** | $M \Rightarrow \neg\ P,\ \neg\ (M \Rightarrow \neg\ \mathbb{S}) \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |
| ***Camenes*** | $P \Rightarrow M,\ M \Rightarrow \neg\ \mathbb{S} \vdash \mathbb{S} \Rightarrow \neg\ P$ |
| ***Dimaris*** | $\neg\ (P \Rightarrow \neg\ M),\ M \Rightarrow \mathbb{S} \vdash \neg\ (\mathbb{S} \Rightarrow \neg\ P)$ |
| ***Fresison*** | $P \Rightarrow \neg\ M,\ \neg\ (M \Rightarrow \neg\ \mathbb{S}) \vdash \neg\ (\mathbb{S} \Rightarrow P)$ |

## 2.3 The Theory syllog3

*aristotle*

| | |
|---|---|
| $\$\boldsymbol{a}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(Term2,\ BOOL)\ BR$ |
| ***Complement*** | $Term2 \rightarrow Term2$ |

| | |
|---|---|
| $\boldsymbol{o}$ | $\$u : (Term2,\ BOOL)\ BR$ |
| $\sim$ | $Complement : Term2 \rightarrow Term2$ |

| | |
|---|---|
| ***Term2*** | $Term2$ |

*Right Infix 300*:

$\boldsymbol{a}$ $\boldsymbol{e}$ $\boldsymbol{i}$ $\boldsymbol{u}$

| | |
|---|---|
| $\boldsymbol{a}$ | $\vdash \forall\ A\ B\bullet\ A\ a\ B \Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow B\ x)$ |
| $\boldsymbol{e}$ | $\vdash \forall\ A\ B\bullet\ A\ e\ B \Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow \neg\ B\ x)$ |
| $\boldsymbol{i}$ | $\vdash \forall\ A\ B\bullet\ A\ i\ B \Leftrightarrow (\exists\ x\bullet\ A\ x \wedge B\ x)$ |
| $\boldsymbol{u}$ | $\vdash \forall\ A\ B\bullet\ A\ o\ B \Leftrightarrow (\exists\ x\bullet\ A\ x \wedge \neg\ B\ x)$ |
| ***Complement*** | $\vdash \forall\ A\ \alpha\bullet \sim A\ \alpha \Leftrightarrow \neg\ A\ \alpha$ |

| | |
|---|---|
| ***e_conv_thm*** | $A\ e\ B \vdash B\ e\ A$ |
| ***i_conv_thm*** | $A\ i\ B \vdash B\ i\ A$ |
| ***a_not_conv_thm*** | $\vdash \exists\ A\ B\bullet\ A\ a\ B \wedge \neg\ B\ a\ A$ |
| ***o_not_conv_thm*** | $\vdash \exists\ A\ B\bullet\ A\ o\ B \wedge \neg\ B\ o\ A$ |
| ***ae_obv_thm*** | $A\ a\ B \vdash A\ e \sim B$ |
| ***ea_obv_thm*** | $A\ e\ B \vdash A\ a \sim B$ |
| ***io_obv_thm*** | $A\ i\ B \vdash A\ o \sim B$ |
| ***oi_obv_thm*** | $A\ o\ B \vdash A\ i \sim B$ |
| ***ao_contrad_thm*** | $\vdash A\ a\ B \Leftrightarrow \neg\ A\ o\ B$ |
| ***ei_contrad_thm*** | $\vdash A\ e\ B \Leftrightarrow \neg\ A\ i\ B$ |
| ***¬ae_contrar_thm*** | $\vdash \neg\ (\forall\ A\ B\bullet\ \neg\ (A\ a\ B \wedge A\ e\ B))$ |

| | |
|---|---|
| $\neg$***IU_thm*** | $\vdash \neg\ (\forall\ A\ B \bullet\ A\ i\ B \vee A\ o\ B)$ |
| $\neg$***ai_subalt_thm*** | $\vdash \neg\ (\forall\ A\ B \bullet\ A\ a\ B \Rightarrow A\ i\ B)$ |
| $\neg$***eo_subalt_thm*** | $\vdash \neg\ (\forall\ A\ B \bullet\ A\ e\ B \Rightarrow A\ o\ B)$ |
| ***Barbara*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ a\ P$ |
| ***Celarent*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Darii*** | $M\ a\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ i\ P$ |
| ***Ferio*** | $M\ e\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesare*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Camestres*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ e\ P$ |
| ***Festino*** | $P\ e\ M, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Baroco*** | $P\ a\ M, \mathbb{S}\ o\ M \vdash \mathbb{S}\ o\ P$ |
| ***Disamis*** | $M\ i\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Datisi*** | $M\ a\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Bocardo*** | $M\ o\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Ferison*** | $M\ e\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Camenes*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ e\ P$ |
| ***Dimaris*** | $P\ i\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fresison*** | $P\ e\ M, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |

## 2.4 The Theory syllog4

*aristotle*

| | |
|---|---|
| ***p*** | $Term3 \rightarrow 'a \rightarrow BOOL$ |
| $\$\boldsymbol{a}$ | $(Term3,\ BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(Term3,\ BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(Term3,\ BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(Term3,\ BOOL)\ BR$ |

| | |
|---|---|
| ***o*** | $\$u : (Term3,\ BOOL)\ BR$ |

| | |
|---|---|
| ***Term3*** | $Term3$ |

*Right Infix 300*:

***a*** ***e*** ***i*** ***u***

| | |
|---|---|
| ***p*** | $\vdash \forall\ A\bullet\ p\ A = (\lambda\ x\bullet\ (let\ (v,\ f) = A\ in\ x = v \vee f\ x))$ |
| ***a*** | $\vdash \forall\ A\ B\bullet\ A\ a\ B \Leftrightarrow (\forall\ x\bullet\ p\ A\ x \Rightarrow p\ B\ x)$ |
| ***e*** | $\vdash \forall\ A\ B\bullet\ A\ e\ B \Leftrightarrow (\forall\ x\bullet\ p\ A\ x \Rightarrow \neg\ p\ B\ x)$ |
| ***i*** | $\vdash \forall\ A\ B\bullet\ A\ i\ B \Leftrightarrow (\exists\ x\bullet\ p\ A\ x \wedge p\ B\ x)$ |
| ***u*** | $\vdash \forall\ A\ B\bullet\ A\ o\ B \Leftrightarrow (\exists\ x\bullet\ p\ A\ x \wedge \neg\ p\ B\ x)$ |

| | |
|---|---|
| ***p_∃_lemma*** | $\vdash \forall\ A\bullet\ \exists\ v\bullet\ p\ A\ v$ |
| ***e_conv_thm*** | $A\ e\ B \vdash B\ e\ A$ |
| ***i_conv_thm*** | $A\ i\ B \vdash B\ i\ A$ |
| ***ai_conv_thm*** | $A\ a\ B \vdash B\ i\ A$ |
| ***eo_conv_thm*** | $A\ e\ B \vdash B\ o\ A$ |
| ***sg_09_thm*** | $\vdash A\ a\ B \Leftrightarrow \neg\ A\ o\ B$ |
| ***sg_10_thm*** | $\vdash A\ e\ B \Leftrightarrow \neg\ A\ i\ B$ |
| ***sg_11_thm*** | $\vdash \neg\ (A\ a\ B \wedge A\ e\ B)$ |
| ***sg_12_thm*** | $\vdash A\ i\ B \vee A\ o\ B$ |
| ***sg_13_thm*** | $A\ a\ B \vdash A\ i\ B$ |
| ***sg_14_thm*** | $A\ e\ B \vdash A\ o\ B$ |
| ***Barbara*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ a\ P$ |
| ***Celarent*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Darii*** | $M\ a\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ i\ P$ |
| ***Ferio*** | $M\ e\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesare*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Camestres*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ e\ P$ |

| | |
|---|---|
| ***Festino*** | $P\ e\ M, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Baroco*** | $P\ a\ M, \mathbb{S}\ o\ M \vdash \mathbb{S}\ o\ P$ |
| ***Disamis*** | $M\ i\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Datisi*** | $M\ a\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Bocardo*** | $M\ o\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Ferison*** | $M\ e\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Camenes*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ e\ P$ |
| ***Dimaris*** | $P\ i\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fresison*** | $P\ e\ M, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Darapti*** | $M\ a\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Felapton*** | $M\ e\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Bramantip*** | $P\ a\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fesapo*** | $P\ e\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Barbari*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ i\ P$ |
| ***Celaront*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesaro*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camestrop*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camenop*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |

## 2.5 The Theory syllog5

*aristotle*

| | |
|---|---|
| $\$\boldsymbol{a}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(Term2,\ BOOL)\ BR$ |
| $\boldsymbol{Complement}$ | $Term2 \rightarrow Term2$ |

| | |
|---|---|
| $\boldsymbol{o}$ | $\$u : (Term2,\ BOOL)\ BR$ |
| $\sim$ | $Complement : Term2 \rightarrow Term2$ |

| | |
|---|---|
| $\boldsymbol{Term2}$ | $Term2$ |

*Right Infix 300*:

$\boldsymbol{a} \quad \boldsymbol{e} \quad \boldsymbol{i} \quad \boldsymbol{u}$

| | |
|---|---|
| $\boldsymbol{a}$ | $\vdash \forall\ A\ B$ |
| | $\bullet\ A\ a\ B$ |
| | $\Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow B\ x) \wedge (\exists\ x\bullet\ A\ x) \wedge (\exists\ x\bullet \neg\ B\ x)$ |
| $\boldsymbol{e}$ | $\vdash \forall\ A\ B$ |
| | $\bullet\ A\ e\ B$ |
| | $\Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow \neg\ B\ x) \wedge (\exists\ x\bullet\ A\ x) \wedge (\exists\ x\bullet\ B\ x)$ |
| $\boldsymbol{i}$ | $\vdash \forall\ A\ B$ |
| | $\bullet\ A\ i\ B$ |
| | $\Leftrightarrow (\exists\ x\bullet\ A\ x \wedge B\ x) \vee \neg\ (\exists\ x\bullet\ A\ x) \vee \neg\ (\exists\ x\bullet\ B\ x)$ |
| $\boldsymbol{u}$ | $\vdash \forall\ A\ B$ |
| | $\bullet\ A\ o\ B$ |
| | $\Leftrightarrow (\exists\ x\bullet\ A\ x \wedge \neg\ B\ x)$ |
| | $\vee \neg\ (\exists\ x\bullet\ A\ x)$ |
| | $\vee \neg\ (\exists\ x\bullet \neg\ B\ x)$ |
| $\boldsymbol{Complement}$ | $\vdash \forall\ A\ \alpha\bullet \sim A\ \alpha \Leftrightarrow \neg\ A\ \alpha$ |

| | |
|---|---|
| ***e_conv_thm*** | $A\ e\ B \vdash B\ e\ A$ |
| ***i_conv_thm*** | $A\ i\ B \vdash B\ i\ A$ |
| ***¬a_conv_thm*** | $\vdash \exists\ A\ B \bullet A\ a\ B \wedge \neg\ B\ a\ A$ |
| ***¬o_conv_thm*** | $\vdash \exists\ A\ B \bullet A\ o\ B \wedge \neg\ B\ o\ A$ |
| ***ai_conv_thm*** | $A\ a\ B \vdash B\ i\ A$ |
| ***eo_conv_thm*** | $A\ e\ B \vdash B\ o\ A$ |
| ***ae_obv_thm*** | $A\ a\ B \vdash A\ e \sim B$ |
| ***ea_obv_thm*** | $A\ e\ B \vdash A\ a \sim B$ |
| ***io_obv_thm*** | $A\ i\ B \vdash A\ o \sim B$ |
| ***oi_obv_thm*** | $A\ o\ B \vdash A\ i \sim B$ |
| ***ao_contrad_thm*** | $\vdash A\ a\ B \Leftrightarrow \neg\ A\ o\ B$ |
| ***ei_contrad_thm*** | $\vdash A\ e\ B \Leftrightarrow \neg\ A\ i\ B$ |
| ***ae_contrar_thm*** | $\vdash \neg\ (A\ a\ B \wedge A\ e\ B)$ |
| ***io_subcont_thm*** | $\vdash A\ i\ B \vee A\ o\ B$ |
| ***ai_subalt_thm*** | $\vdash A\ a\ B \Rightarrow A\ i\ B$ |
| ***eo_subalt_thm*** | $\vdash A\ e\ B \Rightarrow A\ o\ B$ |
| ***Barbara*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ a\ P$ |
| ***Celarent*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Darii*** | $M\ a\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ i\ P$ |
| ***Ferio*** | $M\ e\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesare*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Camestres*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ e\ P$ |
| ***Festino*** | $P\ e\ M, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Baroco*** | $P\ a\ M, \mathbb{S}\ o\ M \vdash \mathbb{S}\ o\ P$ |
| ***Disamis*** | $M\ i\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Datisi*** | $M\ a\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Bocardo*** | $M\ o\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Ferison*** | $M\ e\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Camenes*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ e\ P$ |
| ***Dimaris*** | $P\ i\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fresison*** | $P\ e\ M, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Darapti*** | $M\ a\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Felapton*** | $M\ e\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Bramantip*** | $P\ a\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fesapo*** | $P\ e\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Barbari*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ i\ P$ |
| ***Celaront*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |

| | |
|---|---|
| ***Cesaro*** | $P\ e\ M,\ \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camestrop*** | $P\ a\ M,\ \mathbb{S}\ e\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camenop*** | $P\ a\ M,\ M\ e\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |

## 2.6 The Theory syllog6

*aristotle*

| | |
|---|---|
| $\$\boldsymbol{a}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(Term2,\ BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(Term2,\ BOOL)\ BR$ |
| ***Complement*** | $Term2 \rightarrow Term2$ |

| | |
|---|---|
| $\boldsymbol{o}$ | $\$u : (Term2,\ BOOL)\ BR$ |
| $\sim$ | $Complement : Term2 \rightarrow Term2$ |

| | |
|---|---|
| ***Term2*** | $Term2$ |

*Right Infix 300*:

$\boldsymbol{a}$ $\boldsymbol{e}$ $\boldsymbol{i}$ $\boldsymbol{u}$

| | |
|---|---|
| $\boldsymbol{a}$ | $\vdash \forall\ A\ B\bullet\ A\ a\ B \Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow B\ x) \wedge (\exists\ x\bullet\ A\ x)$ |
| $\boldsymbol{e}$ | $\vdash \forall\ A\ B\bullet\ A\ e\ B \Leftrightarrow (\forall\ x\bullet\ A\ x \Rightarrow \neg\ B\ x)$ |
| $\boldsymbol{i}$ | $\vdash \forall\ A\ B\bullet\ A\ i\ B \Leftrightarrow (\exists\ x\bullet\ A\ x \wedge B\ x)$ |
| $\boldsymbol{u}$ | $\vdash \forall\ A\ B\bullet\ A\ o\ B \Leftrightarrow (\exists\ x\bullet\ A\ x \wedge \neg\ B\ x) \vee \neg\ (\exists\ x\bullet\ A\ x)$ |
| ***Complement*** | $\vdash \forall\ A\ \alpha\bullet \sim A\ \alpha \Leftrightarrow \neg\ A\ \alpha$ |

| | |
|---|---|
| ***e_conv_thm*** | $A\ e\ B \vdash B\ e\ A$ |
| ***i_conv_thm*** | $A\ i\ B \vdash B\ i\ A$ |
| ***ai_conv_thm*** | $A\ a\ B \vdash B\ i\ A$ |
| ***eo_conv_thm*** | $A\ e\ B \vdash B\ o\ A$ |
| ***ae_obv_thm*** | $A\ a\ B \vdash A\ e \sim B$ |
| ***iu_obv_thm*** | $A\ i\ B \vdash A\ o \sim B$ |
| ***ao_contrad_thm*** | $\vdash A\ a\ B \Leftrightarrow \neg\ A\ o\ B$ |
| ***ei_contrad_thm*** | $\vdash A\ e\ B \Leftrightarrow \neg\ A\ i\ B$ |
| ***ae_contrar_thm*** | $\vdash \neg\ (A\ a\ B \wedge A\ e\ B)$ |
| ***io_subcont_thm*** | $\vdash A\ i\ B \vee A\ o\ B$ |
| ***ai_subalt_thm*** | $\vdash A\ a\ B \Rightarrow A\ i\ B$ |

***eo_subalt_thm***

| | |
|---|---|
| | $\vdash A\ e\ B \Rightarrow A\ o\ B$ |
| ***Barbara*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ a\ P$ |
| ***Celarent*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Darii*** | $M\ a\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ i\ P$ |
| ***Ferio*** | $M\ e\ P, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesare*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ e\ P$ |
| ***Camestres*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ e\ P$ |
| ***Festino*** | $P\ e\ M, \mathbb{S}\ i\ M \vdash \mathbb{S}\ o\ P$ |
| ***Baroco*** | $P\ a\ M, \mathbb{S}\ o\ M \vdash \mathbb{S}\ o\ P$ |
| ***Disamis*** | $M\ i\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Datisi*** | $M\ a\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Bocardo*** | $M\ o\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Ferison*** | $M\ e\ P, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Camenes*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ e\ P$ |
| ***Dimaris*** | $P\ i\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fresison*** | $P\ e\ M, M\ i\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Darapti*** | $M\ a\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Felapton*** | $M\ e\ P, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Bramantip*** | $P\ a\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ i\ P$ |
| ***Fesapo*** | $P\ e\ M, M\ a\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |
| ***Barbari*** | $M\ a\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ i\ P$ |
| ***Celaront*** | $M\ e\ P, \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |
| ***Cesaro*** | $P\ e\ M, \mathbb{S}\ a\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camestrop*** | $P\ a\ M, \mathbb{S}\ e\ M \vdash \mathbb{S}\ o\ P$ |
| ***Camenop*** | $P\ a\ M, M\ e\ \mathbb{S} \vdash \mathbb{S}\ o\ P$ |

## 2.7 The Theory modsyllog

*aristotle*

| | |
|---|---|
| $\boldsymbol{p}$ | $MPROP \to W \to I \to BOOL$ |
| $\$\boldsymbol{a}$ | $(MPROP,\ W \to BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(MPROP,\ W \to BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(MPROP,\ W \to BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(MPROP,\ W \to BOOL)\ BR$ |
| $\boldsymbol{actual_world}$ | $W$ |
| $\Diamond$ | $(W \to BOOL) \to BOOL$ |
| $\Box$ | $(W \to BOOL) \to BOOL$ |
| $\Diamond_{\boldsymbol{a}}$ | $(W \to BOOL) \to BOOL$ |
| $\models$ | $(W \to BOOL) \to BOOL$ |
| $\$\neg_{\boldsymbol{m}}$ | $(W \to BOOL) \to W \to BOOL$ |

| | |
|---|---|
| $\boldsymbol{o}$ | $\$u : (MPROP,\ W \to BOOL)\ BR$ |
| $\neg$ | $\$\neg_m : (W \to BOOL) \to W \to BOOL$ |

$\boldsymbol{I}$
$\boldsymbol{W}$

| | |
|---|---|
| $\boldsymbol{MPROP}$ | $MPROP$ |

*Right Infix 300*:

| | |
|---|---|
| | $\boldsymbol{a}\quad \boldsymbol{e}\quad \boldsymbol{i}\quad \boldsymbol{u}$ |
| *Prefix 350*: | $\neg_{\boldsymbol{m}}$ |

| | |
|---|---|
| $\boldsymbol{p}$ | $\vdash \forall\ A\bullet\ p\ A = (\lambda\ w\ x\bullet\ (let\ (v, f) = A\ w\ in\ x = v \vee f\ x))$ |
| $\boldsymbol{a}$ | $\vdash \forall\ A\ B\ w\bullet\ (A\ a\ B)\ w \Leftrightarrow (\forall\ x\bullet\ p\ A\ w\ x \Rightarrow p\ B\ w\ x)$ |
| $\boldsymbol{e}$ | $\vdash \forall\ A\ B\ w\bullet\ (A\ e\ B)\ w \Leftrightarrow (\forall\ x\bullet\ p\ A\ w\ x \Rightarrow \neg\ p\ B\ w\ x)$ |
| $\boldsymbol{i}$ | $\vdash \forall\ A\ B\ w\bullet\ (A\ i\ B)\ w \Leftrightarrow (\exists\ x\bullet\ p\ A\ w\ x \wedge p\ B\ w\ x)$ |
| $\boldsymbol{u}$ | $\vdash \forall\ A\ B\ w\bullet\ (A\ o\ B)\ w \Leftrightarrow (\exists\ x\bullet\ p\ A\ w\ x \wedge \neg\ p\ B\ w\ x)$ |
| $\boldsymbol{actual_world}$ | $\vdash T$ |
| $\Diamond$ | $\vdash \forall\ s\bullet\ \Diamond\ s \Leftrightarrow (\exists\ w\bullet\ s\ w)$ |
| $\Box$ | $\vdash \forall\ s\bullet\ \Box\ s \Leftrightarrow (\forall\ w\bullet\ s\ w)$ |
| $\Diamond_{\boldsymbol{a}}$ | $\vdash \forall\ s\bullet\ \Diamond_a\ s \Leftrightarrow \neg\ (\forall\ w\bullet\ s\ w) \wedge \neg\ (\forall\ w\bullet\ \neg\ s\ w)$ |
| $\models$ | $\vdash \forall\ s\bullet \models s \Leftrightarrow s\ actual_world$ |
| $\neg_{\boldsymbol{m}}$ | $\vdash \forall\ x\bullet\ (\neg\ x) = (\lambda\ w\bullet\ \neg\ x\ w)$ |

***p_∃_lemma*** $\vdash \forall\ A\ w \bullet \exists\ v \bullet\ p\ A\ w\ v$

***□◇_thm*** $\Box\ X \vdash \Diamond\ X$

***□⊨_thm*** $\Box\ X \vdash \models X$

***⊨◇_thm*** $\models X \vdash \Diamond\ X$

***◇$_a$◇_thm*** $\Diamond_a\ X \vdash \Diamond\ X$

***◇$_a$¬□_thm*** $\Diamond_a\ X \vdash \neg\ \Box\ X$

***◇$_a$¬$_m$_thm*** $\vdash \Diamond_a\ X \Leftrightarrow \Diamond_a\ (\neg\ X)$

***e_conv_thm*** $\models (A\ e\ B) \vdash \models (B\ e\ A)$

***i_conv_thm*** $\models (A\ i\ B) \vdash \models (B\ i\ A)$

***□e_conv_thm*** $\Box\ (A\ e\ B) \vdash \Box\ (B\ e\ A)$

***□i_conv_thm*** $\Box\ (A\ i\ B) \vdash \Box\ (B\ i\ A)$

***◇e_conv_thm*** $\Diamond\ (A\ e\ B) \vdash \Diamond\ (B\ e\ A)$

***◇i_conv_thm*** $\Diamond\ (A\ i\ B) \vdash \Diamond\ (B\ i\ A)$

***◇$_a$_e_conv_thm***
$\Diamond_a\ (A\ e\ B) \vdash \Diamond_a\ (B\ e\ A)$

***◇$_a$_i_conv_thm***
$\Diamond_a\ (A\ i\ B) \vdash \Diamond_a\ (B\ i\ A)$

***◇$_a$_ao_conv_thm***
$\vdash \Diamond_a\ (A\ a\ B) \Leftrightarrow \Diamond_a\ (A\ o\ B)$

***◇$_a$_ei_conv_thm***
$\vdash \Diamond_a\ (A\ e\ B) \Leftrightarrow \Diamond_a\ (A\ i\ B)$

***□◇e_conv_thm*** $\Box\ (A\ e\ B) \vdash \Diamond\ (B\ e\ A)$

***□⊨e_conv_thm*** $\Box\ (A\ e\ B) \vdash \models (B\ e\ A)$

***⊨ai_conv_thm*** $\models (A\ a\ B) \vdash \models (B\ i\ A)$

***⊨eo_conv_thm*** $\models (A\ e\ B) \vdash \models (B\ o\ A)$

***□◇ai_conv_thm***
$\Box\ (A\ a\ B) \vdash \Diamond\ (B\ i\ A)$

***□◇eo_conv_thm***
$\Box\ (A\ e\ B) \vdash \Diamond\ (B\ o\ A)$

***mod_gen_□□□*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Box\ FP \wedge \Box\ SP \Rightarrow \Box\ CS$

***mod_gen_□□◇*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Box\ FP \wedge \Box\ SP \Rightarrow \Diamond\ CS$

***mod_gen_□□⊨*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Box\ FP \wedge \Box\ SP \Rightarrow \models CS$

***mod_gen_◇□◇*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Diamond\ FP \wedge \Box\ SP \Rightarrow \Diamond\ CS$

***mod_gen_□◇◇*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Box\ FP \wedge \Diamond\ SP \Rightarrow \Diamond\ CS$

***mod_gen_□⊨⊨*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \Box\ FP \wedge \models SP \Rightarrow \models CS$

***mod_gen_⊨□⊨*** $\vdash \forall\ FP\ SP\ CS$
$\bullet\ (\forall\ w \bullet\ FP\ w \wedge SP\ w \Rightarrow CS\ w) \Rightarrow \models FP \wedge \Box\ SP \Rightarrow \models CS$

$$\boldsymbol{mod_gen_\models\models\models} \quad \vdash \forall\ FP\ \ SP\ \ CS \bullet (\forall\ w \bullet FP\ \ w \wedge SP\ \ w \Rightarrow CS\ \ w) \Rightarrow \models FP \wedge \models SP \Rightarrow \models CS$$

## 2.8 The Theory ariscat

*aristotle*

| | |
|---|---|
| ***CatSubs*** | $CAT$ |
| ***Cat*** | $CATM \rightarrow CAT$ |
| ***IndvSet*** | $CATM \rightarrow ISUB\ \mathbb{P}$ |
| ***AttrSet*** | $CATM \rightarrow (ISUB \rightarrow BOOL)\ \mathbb{P}$ |
| ***CatSet*** | $CATM \rightarrow (ISUB + (ISUB \rightarrow BOOL))\ \mathbb{P}$ |
| $***izz*** | $(CATM,\ BOOL)\ BR$ |
| $***hazz*** | $(CATM,\ BOOL)\ BR$ |
| $***essentially_predicable_of*** | $(CATM,\ BOOL)\ BR$ |
| $***accidentally_predicable_of*** | $(CATM,\ BOOL)\ BR$ |
| $***predicable_of*** | $(CATM,\ BOOL)\ BR$ |
| ***individual*** | $CATM \rightarrow BOOL$ |
| ***particular*** | $CATM \rightarrow BOOL$ |
| ***universal*** | $CATM \rightarrow BOOL$ |
| ***SomeThing*** | $CATM \rightarrow BOOL$ |
| ***form*** | $CATM \rightarrow BOOL$ |

***ACAT***
***ISUB***

| | |
|---|---|
| ***CATM*** | $CATM$ |
| ***CAT*** | $CAT$ |

*Right Infix 300*:

***accidentally_predicable_of*** ***izz***
***essentially_predicable_ofpredicable_of***
***hazz***

***CatSubs*** $\vdash CatSubs = InL\ One$

***Cat*** $\vdash \forall\ x$
$\bullet\ Cat\ x$
$= (if\ IsL\ x\ then\ CatSubs\ else\ InR\ (Fst\ (OutR\ x)))$

***IndvSet*** $\vdash \forall\ x \bullet\ IndvSet\ x = OutL\ x$

***AttrSet*** $\vdash \forall\ x \bullet\ AttrSet\ x = Snd\ (OutR\ x)$

***CatSet*** $\vdash \forall\ x$
$\bullet\ CatSet\ x$
$= (if\ Cat\ x = CatSubs$
$then\ \{y | \exists\ z \bullet\ z \in IndvSet\ x \wedge y = InL\ z\}$
$else\ \{y | \exists\ z \bullet\ z \in AttrSet\ x \wedge y = InR\ z\})$

***izz*** $\vdash \forall\ A\ B \bullet\ A\ izz\ B \Leftrightarrow Cat\ A = Cat\ B \wedge CatSet\ A \subseteq CatSet\ B$

***hazz*** $\vdash \forall\ A\ B$
$\bullet\ A\ hazz\ B$
$\Leftrightarrow Cat\ A = CatSubs$
$\wedge \neg\ Cat\ B = CatSubs$
$\wedge (\exists\ a$
$\bullet\ a \in AttrSet\ B \wedge (\forall\ s \bullet\ s \in IndvSet\ A \Rightarrow a\ s))$

***essentially_predicable_of***
$\vdash \forall\ A\ B \bullet\ A\ essentially_predicable_of\ B \Leftrightarrow B\ izz\ A$

***accidentally_predicable_of***
$\vdash \forall\ A\ B \bullet\ A\ accidentally_predicable_of\ B \Leftrightarrow B\ hazz\ A$

***predicable_of***
$\vdash \forall\ A\ B$
$\bullet\ A\ predicable_of\ B$
$\Leftrightarrow A\ essentially_predicable_of\ B$
$\vee\ A\ accidentally_predicable_of\ B$

***individual*** $\vdash \forall\ A \bullet\ individual\ A \Leftrightarrow (\exists\ a \bullet\ CatSet\ A = \{a\})$

***particular*** $\vdash \forall\ A \bullet\ particular\ A \Leftrightarrow individual\ A \wedge Cat\ A = CatSubs$

***universal*** $\vdash \forall\ A \bullet\ universal\ A \Leftrightarrow \neg\ particular\ A$

***SomeThing*** $\vdash \forall\ x \bullet\ SomeThing\ x \Rightarrow individual\ x$

***form*** $\vdash \forall\ x \bullet\ form\ x \Rightarrow SomeThing\ x \wedge universal\ x$

***catm_eq_lemma***
$\vdash \forall\ A\ B \bullet\ A = B \Leftrightarrow Cat\ A = Cat\ B \wedge CatSet\ A = CatSet\ B$

***izz_lemma1*** $\vdash \forall\ A\ B$
$\bullet\ Cat\ A = CatSubs$
$\Rightarrow (A\ izz\ B$
$\Leftrightarrow Cat\ B = CatSubs \wedge IndvSet\ A \subseteq IndvSet\ B)$

***izz_lemma2*** $\vdash \forall\ A\ B$
$\bullet\ Cat\ B = CatSubs$
$\Rightarrow (A\ izz\ B$
$\Leftrightarrow Cat\ A = CatSubs \wedge IndvSet\ A \subseteq IndvSet\ B)$

***izz_lemma3*** $\vdash \forall\ A\ B$
$\bullet\ \neg\ Cat\ A = CatSubs$
$\Rightarrow (A\ izz\ B$
$\Leftrightarrow Cat\ B = Cat\ A \wedge AttrSet\ A \subseteq AttrSet\ B)$

***izz_lemma4*** $\vdash \forall\ A\ B$
$\bullet\ \neg\ Cat\ B = CatSubs$
$\Rightarrow (A\ izz\ B$
$\Leftrightarrow Cat\ B = Cat\ A \wedge AttrSet\ A \subseteq AttrSet\ B)$

***l01*** $\vdash \forall\ A\bullet\ A\ izz\ A$

***l01b*** $\vdash \forall\ A\bullet\ \neg\ A\ hazz\ A$

***l02*** $\vdash \forall\ A\ B\ C\bullet\ A\ izz\ B \wedge B\ izz\ C \Rightarrow A\ izz\ C$

***l03*** $\vdash \forall\ A\ B\bullet\ A\ hazz\ B \Rightarrow \neg\ A\ izz\ B$

***l03b*** $\vdash \forall\ A\ B\bullet\ A\ izz\ B \Rightarrow \neg\ A\ hazz\ B$

***l03c*** $\vdash \forall\ A\ B\bullet\ A\ hazz\ B \Rightarrow \neg\ A = B$

***l04a*** $\vdash \forall\ A\ B\ C\bullet\ A\ hazz\ B \wedge B\ izz\ C \Rightarrow A\ hazz\ C$

***l04*** $\vdash \forall\ A\ B\bullet\ A\ hazz\ B \Leftrightarrow (\exists\ C\bullet\ A\ hazz\ C \wedge C\ izz\ B)$

***l04b*** $\vdash \forall\ A\ B\ C\bullet\ A\ izz\ B \wedge B\ hazz\ C \Rightarrow A\ hazz\ C$

***l06n*** $\vdash \neg\ (\forall\ A\ B$
$\bullet\ A\ hazz\ B \wedge particular\ A \Rightarrow (\exists\ C\bullet\ C \neq A \wedge A\ izz\ B))$

***l07*** $\vdash \forall\ A\ B$
$\bullet\ A\ predicable_of\ B$
$\Leftrightarrow B\ izz\ A \vee (\exists\ C\bullet\ B\ hazz\ C \wedge C\ izz\ A)$

***l08*** $\vdash \forall\ A\bullet\ A\ essentially_predicable_of\ B \Leftrightarrow B\ izz\ A$

***l09*** $\vdash \forall\ A$
$\bullet\ A\ accidentally_predicable_of\ B$
$\Leftrightarrow (\exists\ C\bullet\ B\ hazz\ C \wedge C\ izz\ A)$

***l10*** $\vdash \forall\ A\ B\bullet\ A = B \Leftrightarrow A\ izz\ B \wedge B\ izz\ A$

***l16*** $\vdash \forall\ A\ B$
$\bullet\ A\ predicable_of\ B$
$\Leftrightarrow B\ izz\ A \vee (\exists\ C\bullet\ B\ hazz\ C \wedge C\ izz\ A)$

***l17*** $\vdash \forall\ A\bullet\ A\ essentially_predicable_of\ A$

***l18*** $\vdash \forall\ A\bullet\ A\ accidentally_predicable_of\ B \Rightarrow A \neq B$

***l19*** $\vdash \forall\ A\bullet\ \neg\ A\ accidentally_predicable_of\ A$

***l20*** $\vdash \forall\ A\bullet\ particular\ A \Rightarrow individual\ A$

***l22*** $\vdash \neg\ (\exists\ A\bullet\ particular\ A \wedge form\ A)$

***l23b*** $\vdash \forall\ A\bullet\ form\ A \Rightarrow individual\ A$

***l24a*** $\vdash \forall\ A\ B$
$\bullet\ particular\ A \wedge individual\ B \wedge A\ izz\ B$
$\Rightarrow particular\ B$

***l24*** $\vdash \forall\ A\bullet\ particular\ A \Rightarrow \neg\ (\exists\ B\bullet\ form\ B \wedge A\ izz\ B)$

***l26*** $\vdash \forall\ A\ B$
$\bullet\ form\ A \wedge particular\ B$
$\Rightarrow (A\ predicable_of\ B \Leftrightarrow B\ hazz\ A)$

***l05b*** $\vdash \forall\ x \bullet\ form\ x \Rightarrow universal\ x$

## 2.9 The Theory syllmetap

| | |
|---|---|
| | *aristotle* |
| | *gccon* |
| ***Category_of_Substance*** | |
| | *CAT* |
| ***actual_world*** | *W* |
| ***tag*** | $I \to TAG$ |
| ***category*** | $I \to CAT$ |
| ***mk_SubsTerm*** | $TAG\ \mathbb{P} \to TermM$ |
| ***mk_AttrTerm*** | $CAT \times TAG\ \mathbb{P} \to TermM$ |
| ***Substance*** | *TermM* |
| $***not*** | $COPULA \to COPULA$ |
| $***InTermM*** | $I \to TermM \to BOOL$ |
| ***All*** | $TermM \to COPULA \to TermM \to MPROP$ |
| ***Some*** | $TermM \to COPULA \to TermM \to MPROP$ |
| ***izz*** | *COPULA* |
| ***hazz*** | *COPULA* |
| ***are*** | *COPULA* |
| $\Diamond$ | $MPROP \to MPROP$ |
| $\Box$ | $MPROP \to MPROP$ |
| $\neg_{\boldsymbol{a}}$ | $MPROP \to MPROP$ |
| $\$\wedge_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\Rightarrow_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\Leftrightarrow_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\forall_{\boldsymbol{a}}$ | $(TermM \to MPROP) \to MPROP$ |
| $\$\exists_{\boldsymbol{a}}$ | $(TermM \to MPROP) \to MPROP$ |
| $\$\models$ | $MPROP\ LIST \to MPROP \to BOOL$ |
| $\$\Vdash$ | $MPROP\ LIST \to MPROP \to BOOL$ |

| | |
|---|---|
| $\neg$ | $\neg_a : MPROP \to MPROP$ |
| $\wedge$ | $\$\wedge_a : (MPROP,\ MPROP)\ BR$ |
| $\Rightarrow$ | $\$\Rightarrow_a : (MPROP,\ MPROP)\ BR$ |
| $\Leftrightarrow$ | $\$\Leftrightarrow_a : (MPROP,\ MPROP)\ BR$ |
| $\forall$ | $\$\forall_a : (TermM \to MPROP) \to MPROP$ |
| $\exists$ | $\$\exists_a : (TermM \to MPROP) \to MPROP$ |

***ACAT***
***TAG***

| | |
|---|---|
| ***CAT*** | *CAT* |
| ***I*** | *I* |
| ***W*** | *W* |
| ***TermM*** | *TermM* |
| ***COPULA*** | *COPULA* |
| ***MPROP*** | *MPROP* |

| | |
|---|---|
| *Binder*: | $\forall_{\boldsymbol{a}}$ $\exists_{\boldsymbol{a}}$ |
| *Right Infix 100*: | $\Vdash$ $\models$ |
| *Right Infix 200*: | $\Leftrightarrow_{\boldsymbol{a}}$ |
| *Right Infix 210*: | $\Rightarrow_{\boldsymbol{a}}$ |
| *Right Infix 220*: | $\wedge_{\boldsymbol{a}}$ |
| *Right Infix 300*: | ***InTermM*** |
| *Postfix 100*: | ***not*** |

| | |
|---|---|
| ***Category_of_Substance*** | $\vdash$ *Category_of_Substance* = *InL One* |
| ***actual_world*** | $\vdash$ *T* |
| ***category*** | |
| ***tag*** | $\vdash \forall$ *ct*• *category ct* = *Fst ct* $\wedge$ *tag ct* = *Snd ct* |
| ***mk_SubsTerm*** | $\vdash \forall$ *s*• *mk_SubsTerm s* = (*Category_of_Substance*, *NeSet s*) |
| ***mk_AttrTerm*** | $\vdash \forall$ *s*• *mk_AttrTerm s* = (*Fst s*, *NeSet* (*Snd s*)) |
| ***Substance*** | $\vdash$ *Substance* = *mk_SubsTerm Universe* |
| ***not*** | $\vdash \forall$ *pred*• (*pred not*) = ($\lambda$ *pa t w*• $\neg$ *pred pa t w*) |
| ***InTermM*** | $\vdash \forall$ *c1t c2ts*<br>• *c1t InTermM c2ts*<br>$\Leftrightarrow$ *Fst c1t* = *Fst c2ts* $\wedge$ *Snd c1t* $\in$ *PeSet* (*Snd c2ts*) |
| ***All*** | $\vdash \forall$ *s r p*<br>• *All s r p* = ($\lambda$ *w*• $\forall$ *z*• *z InTermM s* $\Rightarrow$ *r z p w*) |
| ***Some*** | $\vdash \forall$ *s r p*<br>• *Some s r p* = ($\lambda$ *w*• $\exists$ *z*• *z InTermM s* $\wedge$ *r z p w*) |
| ***izz*** | $\vdash \forall$ *j t*• *izz j t* = ($\lambda$ *w*• *j InTermM t*) |
| ***hazz*** | $\vdash \forall$ *c1t c2ts*<br>• *hazz c1t c2ts*<br>= ($\lambda$ *w*<br>• *Fst c1t* = *Category_of_Substance*<br>$\wedge$ $\neg$ *Fst c2ts* = *Category_of_Substance*<br>$\wedge$ ($\exists$ *b* |

$\bullet\ b \in PeSet\ (Snd\ c2ts)$
$\wedge\ Snd\ c1t \in w\ (OutR\ (Fst\ c2ts),\ b)))$

***are*** $\vdash \forall\ pa\ t\bullet\ are\ pa\ t = (\lambda\ w\bullet\ izz\ pa\ t\ w \vee hazz\ pa\ t\ w)$

$\Diamond$ $\vdash \forall\ p\bullet\ \Diamond\ p = (\lambda\ w\bullet\ \exists\ w'\bullet\ p\ w')$

$\Box$ $\vdash \forall\ p\bullet\ \Box\ p = (\lambda\ w\bullet\ \forall\ w'\bullet\ p\ w')$

$\neg_{\boldsymbol{a}}$ $\vdash \forall\ p\bullet\ (\neg\ p) = (\lambda\ w\bullet\ \neg\ p\ w)$

$\wedge_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet\ (p \wedge q) = (\lambda\ w\bullet\ p\ w \wedge q\ w)$

$\Rightarrow_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet\ (p \Rightarrow q) = (\lambda\ w\bullet\ p\ w \Rightarrow q\ w)$

$\Leftrightarrow_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet\ (p \Leftrightarrow q) = (\lambda\ w\bullet\ p\ w \Leftrightarrow q\ w)$

$\forall_{\boldsymbol{a}}$ $\vdash \forall\ mpf\bullet\ \$\forall\ mpf = (\lambda\ w\bullet\ \forall\ t\bullet\ mpf\ t\ w)$

$\exists_{\boldsymbol{a}}$ $\vdash \forall\ mpf\bullet\ \$\exists\ mpf = (\lambda\ w\bullet\ \exists\ t\bullet\ mpf\ t\ w)$

$\models$ $\vdash \forall\ lp\ c$
$\bullet\ lp \models c$
$\Leftrightarrow Fold\ (\lambda\ p\ t\bullet\ p\ actual_world \wedge t)\ lp\ T$
$\Rightarrow c\ actual_world$

$\Vdash$ $\vdash \forall\ lp\ c$
$\bullet\ lp \Vdash c \Leftrightarrow (\forall\ w\bullet\ Fold\ (\lambda\ p\ t\bullet\ p\ w \wedge t)\ lp\ T \Rightarrow c\ w)$

***interm_∃_lemma***
$\vdash \forall\ t\bullet\ \exists\ j\bullet\ j\ InTermM\ t$

***not_izz_and_hazz_lemma1***
$\vdash \forall\ pa\ t\ w\bullet\ \neg\ (izz\ pa\ t\ w \wedge hazz\ pa\ t\ w)$

***are_izz_neq_hazz_lemma***
$\vdash \forall\ pa\ t\ w\bullet\ are\ pa\ t\ w \Leftrightarrow \neg\ (izz\ pa\ t\ w \Leftrightarrow hazz\ pa\ t\ w)$

***All_are_izz_or_hazz_lemma***
$\vdash \forall\ A\ B\ w$
$\bullet\ All\ A\ are\ B\ w \Leftrightarrow All\ A\ izz\ B\ w \vee All\ A\ hazz\ B\ w$

***Some_are_izz_or_hazz_lemma***
$\vdash \forall\ A\ B\ w$
$\bullet\ Some\ A\ are\ B\ w \Leftrightarrow Some\ A\ izz\ B\ w \vee Some\ A\ hazz\ B\ w$

***All_are_not_lemma***
$\vdash \forall\ A\ B\ w$
$\bullet\ All\ A\ (are\ not)\ B\ w$
$\Leftrightarrow All\ A\ (izz\ not)\ B\ w \wedge All\ A\ (hazz\ not)\ B\ w$

***izz_not_lemma***
$\vdash All\ B\ (izz\ not)\ A = All\ A\ (izz\ not)\ B$

***some_izz_lemma***
$\vdash Some\ B\ izz\ A = Some\ A\ izz\ B$

***izz_conv1*** $\vdash [All\ B\ (izz\ not)\ A] \Vdash All\ A\ (izz\ not)\ B$

***izz_conv2*** $\vdash [All\ B\ izz\ A] \Vdash Some\ A\ izz\ B$

***izz_conv3*** $\vdash [Some\ B\ izz\ A] \Vdash Some\ A\ izz\ B$

***not_hazz_not_lemma***
$\vdash \neg\ (\forall\ A\ B\bullet\ All\ B\ (hazz\ not)\ A = All\ A\ (hazz\ not)\ B)$

***not_some_hazz_lemma***
$\vdash \neg\ (\forall\ A\ B\bullet\ Some\ B\ hazz\ A = Some\ A\ hazz\ B)$

***hazz_conv2*** $\vdash [All\ A\ hazz\ B] \Vdash Some\ A\ hazz\ B$

***◇_conv*** $\vdash [P] \Vdash Q \Rightarrow [\Diamond\ P] \Vdash \Diamond\ Q$

***□_conv*** $\vdash [P] \Vdash Q \Rightarrow [\Box\ P] \Vdash \Box\ Q$

***□_izz_thm*** $\vdash [\Box\ (All\ A\ izz\ B)] \vDash All\ A\ izz\ B$

***□_hazz_thm*** $\vdash [\Box\ (All\ A\ hazz\ B)] \vDash All\ A\ izz\ B$

***izz_□_thm*** $\vdash [All\ A\ izz\ B] \vDash \Box\ (All\ A\ izz\ B)$

***not_□_hazz_thm***
$\vdash [] \vDash (\neg\ \Box\ (All\ A\ hazz\ B))$

***□_izz_thm2*** $\vdash [\Box\ (All\ A\ izz\ B)] \Vdash All\ A\ izz\ B$

***□_hazz_thm2*** $\vdash [\Box\ (All\ A\ hazz\ B)] \Vdash All\ A\ izz\ B$

***izz_□_thm2*** $\vdash [All\ A\ izz\ B] \Vdash \Box\ (All\ A\ izz\ B)$

***not_□_hazz_thm2***
$\vdash [] \Vdash (\neg\ \Box\ (All\ A\ hazz\ B))$

***□_elim_thm*** $\vdash [\Box\ P] \vDash P$

***□_intro_thm*** $\vdash [P] \vDash \Diamond\ P$

***□_◇_thm*** $\vdash [\Box\ P] \vDash \Diamond\ P$

***¬_All_conv_thm***
$\vdash (\neg\ All\ A\ cop\ B) = Some\ A\ (cop\ not)\ B$

***¬_All_not_conv_thm2***
$\vdash (\neg\ All\ A\ (cop\ not)\ B) = Some\ A\ cop\ B$

***¬_Some_conv_thm***
$\vdash (\neg\ Some\ A\ cop\ B) = All\ A\ (cop\ not)\ B$

***¬_Some_not_conv_thm***
$\vdash (\neg\ Some\ A\ (cop\ not)\ B) = All\ A\ cop\ B$

***Barbara_izz*** $\vdash [All\ M\ izz\ P;\ All\ \mathbb{S}\ izz\ M] \Vdash All\ \mathbb{S}\ izz\ P$

***Celarent_izz*** $\vdash [All\ M\ (izz\ not)\ P;\ All\ \mathbb{S}\ izz\ M] \Vdash All\ \mathbb{S}\ (izz\ not)\ P$

***Darii_izz*** $\vdash [All\ M\ izz\ P;\ Some\ \mathbb{S}\ izz\ M] \Vdash Some\ \mathbb{S}\ izz\ P$

***Ferio_izz*** $\vdash [All\ M\ (izz\ not)\ P;\ Some\ \mathbb{S}\ izz\ M]$
$\Vdash Some\ \mathbb{S}\ (izz\ not)\ P$

***Cesare_izz*** $\vdash [All\ P\ (izz\ not)\ M;\ All\ \mathbb{S}\ izz\ M] \Vdash All\ \mathbb{S}\ (izz\ not)\ P$

***Camestres_izz***
$\vdash [All\ P\ izz\ M;\ All\ \mathbb{S}\ (izz\ not)\ M] \Vdash All\ \mathbb{S}\ (izz\ not)\ P$

***Festino_izz*** $\vdash [All\ P\ (izz\ not)\ M;\ Some\ \mathbb{S}\ izz\ M]$
$\Vdash Some\ \mathbb{S}\ (izz\ not)\ P$

***Baroco_izz*** $\vdash [All\ P\ izz\ M;\ Some\ \mathbb{S}\ (izz\ not)\ M]$
$\Vdash Some\ \mathbb{S}\ (izz\ not)\ P$

***Disamis_izz*** $\vdash [Some\ M\ izz\ P;\ All\ M\ izz\ \mathbb{S}] \Vdash Some\ \mathbb{S}\ izz\ P$

***Datisi_izz*** $\vdash [All\ M\ izz\ P;\ Some\ M\ izz\ \mathbb{S}] \Vdash Some\ \mathbb{S}\ izz\ P$

***Bocardo_izz*** $\vdash [Some\ M\ (izz\ not)\ P;\ All\ M\ izz\ \mathbb{S}]$
$\Vdash Some\ \mathbb{S}\ (izz\ not)\ P$

***Ferison_izz*** $\vdash [All\ M\ (izz\ not)\ P;\ Some\ M\ izz\ \mathbb{S}]$

$\Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Camenes_izz*** $\vdash [\mathit{All}\ P\ \mathit{izz}\ M;\ \mathit{All}\ M\ (\mathit{izz}\ \mathit{not})\ \mathbb{S}] \Vdash \mathit{All}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Dimaris_izz*** $\vdash [\mathit{Some}\ P\ \mathit{izz}\ M;\ \mathit{All}\ M\ \mathit{izz}\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ \mathit{izz}\ P$

***Fresison_izz*** $\vdash [\mathit{All}\ P\ (\mathit{izz}\ \mathit{not})\ M;\ \mathit{Some}\ M\ \mathit{izz}\ \mathbb{S}]$

$\Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Darapti_izz*** $\vdash [\mathit{All}\ M\ \mathit{izz}\ P;\ \mathit{All}\ M\ \mathit{izz}\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ \mathit{izz}\ P$

***Felapton_izz*** $\vdash [\mathit{All}\ M\ (\mathit{izz}\ \mathit{not})\ P;\ \mathit{All}\ M\ \mathit{izz}\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Bramantip_izz***

$\vdash [\mathit{All}\ P\ \mathit{izz}\ M;\ \mathit{All}\ M\ \mathit{izz}\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ \mathit{izz}\ P$

***Fesapo_izz*** $\vdash [\mathit{All}\ P\ (\mathit{izz}\ \mathit{not})\ M;\ \mathit{All}\ M\ \mathit{izz}\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Barbari_izz*** $\vdash [\mathit{All}\ M\ \mathit{izz}\ P;\ \mathit{All}\ \mathbb{S}\ \mathit{izz}\ M] \Vdash \mathit{Some}\ \mathbb{S}\ \mathit{izz}\ P$

***Celaront_izz*** $\vdash [\mathit{All}\ M\ (\mathit{izz}\ \mathit{not})\ P;\ \mathit{All}\ \mathbb{S}\ \mathit{izz}\ M] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Cesaro_izz*** $\vdash [\mathit{All}\ P\ (\mathit{izz}\ \mathit{not})\ M;\ \mathit{All}\ \mathbb{S}\ \mathit{izz}\ M] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Camestrop_izz***

$\vdash [\mathit{All}\ P\ \mathit{izz}\ M;\ \mathit{All}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ M] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

***Camenop_izz*** $\vdash [\mathit{All}\ P\ \mathit{izz}\ M;\ \mathit{All}\ M\ (\mathit{izz}\ \mathit{not})\ \mathbb{S}] \Vdash \mathit{Some}\ \mathbb{S}\ (\mathit{izz}\ \mathit{not})\ P$

## 2.10 The Theory gccon

| | |
|---|---|
| | *syllmetap* |
| ***individual*** | $TermM \rightarrow BOOL$ |
| ***Mp*** | $BOOL \rightarrow MPROP$ |
| ***Individual*** | $TermM \rightarrow MPROP$ |
| ***particular*** | $TermM \rightarrow MPROP$ |
| ***substantial*** | $TermM \rightarrow MPROP$ |
| ***universal*** | $TermM \rightarrow MPROP$ |

***individual*** $\vdash \forall\ A\bullet\ individual\ A \Leftrightarrow (\exists\ b\bullet\ Snd\ A = NeSet\ \{b\})$

***Mp*** $\vdash \forall\ p\bullet\ Mp\ p = (\lambda\ w\bullet\ p)$

***Individual***
$$\vdash \forall\ A \bullet\ Individual\ A = \Box\ (\forall\ B\bullet\ All\ B\ izz\ A \Rightarrow All\ A\ izz\ B)$$

***particular***
$$\vdash \forall\ A \bullet\ particular\ A = \Box\ (\forall\ B\bullet\ All\ B\ are\ A \Rightarrow All\ A\ izz\ B \wedge All\ B\ izz\ A)$$

***substantial***
$$\vdash \forall\ A \bullet\ substantial\ A = \Box\ (\forall\ B\bullet\ All\ B\ are\ A \Rightarrow All\ B\ izz\ A)$$

***universal***
$$\vdash \forall\ A \bullet\ universal\ A = \Diamond\ (\exists\ B \bullet\ All\ B\ are\ A \wedge \neg\ (All\ A\ izz\ B \wedge All\ B\ izz\ A))$$

***FP1*** $\vdash [] \Vdash All\ A\ izz\ A$

***FP2*** $\vdash [All\ A\ izz\ B;\ All\ B\ izz\ C] \Vdash All\ A\ izz\ C$

***FP3*** $\vdash [All\ A\ hazz\ B] \Vdash (\neg\ All\ A\ izz\ B)$

***FP3b*** $\vdash [All\ A\ hazz\ B] \Vdash Some\ A\ (izz\ not)\ B$

***FP4a*** $\vdash [All\ A\ hazz\ B;\ All\ B\ izz\ C] \Vdash All\ A\ hazz\ C$

***FP4***
$$\vdash [] \Vdash (All\ A\ hazz\ B \Leftrightarrow (\exists\ C\bullet\ All\ A\ hazz\ C \wedge All\ C\ izz\ B))$$

***FP4c***
$$\vdash [Mp\ (individual\ A)] \Vdash (All\ A\ hazz\ B \Leftrightarrow (\exists\ C \bullet\ Mp\ (individual\ C) \wedge All\ A\ hazz\ C \wedge All\ C\ izz\ B))$$

***individual_lemma*1**

$$\vdash \forall\ A\bullet\ Individual\ A = (\forall\ B\bullet\ All\ B\ izz\ A \Rightarrow All\ A\ izz\ B)$$

***individual_lemma*2**

$$\vdash Individual\ A = Mp\ (individual\ A)$$

# Chapter 3

# Leibniz

## 3.1 The Theory leibniz01

*misc2*

***leibniz_identity***

$$\vdash \forall\ x\ y\bullet\ (\forall\ P\bullet\ P\ x \Leftrightarrow P\ y) \Rightarrow x = y$$

## 3.2 The Theory leibniz02

*aristotle*

| | |
|---|---|
| ***Some*** | $TTV \rightarrow BOOL$ |
| ***IsPos*** | $TTV \rightarrow BOOL$ |
| $\$\boldsymbol{a}$ | $(TermL,\ BOOL)\ BR$ |
| $\$\boldsymbol{i}$ | $(TermL,\ BOOL)\ BR$ |
| $\$\boldsymbol{e}$ | $(TermL,\ BOOL)\ BR$ |
| $\$\boldsymbol{u}$ | $(TermL,\ BOOL)\ BR$ |
| ***Complement*** | $TermL \rightarrow TermL$ |

| | |
|---|---|
| $\boldsymbol{o}$ | $\$u : (TermL,\ BOOL)\ BR$ |
| $\sim$ | $Complement : TermL \rightarrow TermL$ |

| | |
|---|---|
| ***TermL*** | $TermL$ |

*Right Infix 300*:

$\boldsymbol{a}\quad \boldsymbol{e}\quad \boldsymbol{i}\quad \boldsymbol{u}$

| | |
|---|---|
| ***Some*** | $\vdash \forall\ x\bullet\ Some\ x \Leftrightarrow \neg\ x = pU$ |
| ***IsPos*** | $\vdash \forall\ x\bullet\ IsPos\ x \Leftrightarrow x = pTrue$ |
| $\boldsymbol{a}$ | $\vdash \forall\ A\ B$ |
| | $\bullet\ A\ a\ B$ |
| | $\Leftrightarrow (\forall\ x$ |
| | $\bullet\ (B\ x = pTrue \Rightarrow A\ x = pTrue)$ |
| | $\wedge\ (B\ x = pFalse \Rightarrow A\ x = pFalse))$ |
| $\boldsymbol{i}$ | $\vdash \forall\ A\ B\bullet\ A\ i\ B \Leftrightarrow (\forall\ x\bullet\ A\ x = B\ x \Rightarrow A\ x = pU)$ |
| $\boldsymbol{e}$ | $\vdash \forall\ A\ B\bullet\ A\ e\ B \Leftrightarrow \neg\ A\ i\ B$ |
| $\boldsymbol{u}$ | $\vdash \forall\ A\ B\bullet\ A\ o\ B \Leftrightarrow \neg\ A\ a\ B$ |
| ***Complement*** | $\vdash \forall\ A\ \alpha$ |
| | $\bullet \sim A\ \alpha$ |
| | $= (if\ A\ \alpha = pTrue$ |
| | $then\ pFalse$ |
| | $else\ if\ A\ \alpha = pFalse$ |
| | $then\ pTrue$ |
| | $else\ pU)$ |

| | |
|---|---|
| ***e_conv_thm*** | $A\ e\ B \vdash B\ e\ A$ |
| ***i_conv_thm*** | $A\ i\ B \vdash B\ i\ A$ |

## 3.3 The Theory leibniz03

*misc2*

| | |
|---|---|
| ***extension*** | $PRED \to W$ |
| ***actual_world*** | $W$ |
| $\$\boldsymbol{not}$ | $COPULA \to COPULA$ |
| ***All*** | $PRED \to COPULA \to PRED \to MPROP$ |
| ***Some*** | $PRED \to COPULA \to PRED \to MPROP$ |
| ***are*** | $COPULA$ |
| $\Diamond$ | $MPROP \to MPROP$ |
| $\Box$ | $MPROP \to MPROP$ |
| $\neg_{\boldsymbol{a}}$ | $MPROP \to MPROP$ |
| $\$\wedge_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\Rightarrow_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\Leftrightarrow_{\boldsymbol{a}}$ | $(MPROP,\ MPROP)\ BR$ |
| $\$\forall_{\boldsymbol{a}}$ | $(PRED \to MPROP) \to MPROP$ |
| $\$\exists_{\boldsymbol{a}}$ | $(PRED \to MPROP) \to MPROP$ |
| $\$\models$ | $MPROP\ LIST \to MPROP \to BOOL$ |
| $\$\Vdash$ | $MPROP\ LIST \to MPROP \to BOOL$ |
| ***individual*** | $PRED \to MPROP$ |

| | |
|---|---|
| $\neg$ | $\neg_a : MPROP \to MPROP$ |
| $\wedge$ | $\$\wedge_a : (MPROP,\ MPROP)\ BR$ |
| $\Rightarrow$ | $\$\Rightarrow_a : (MPROP,\ MPROP)\ BR$ |
| $\Leftrightarrow$ | $\$\Leftrightarrow_a : (MPROP,\ MPROP)\ BR$ |
| $\forall$ | $\$\forall_a : (PRED \to MPROP) \to MPROP$ |
| $\exists$ | $\$\exists_a : (PRED \to MPROP) \to MPROP$ |

***ISUB***
***PRED***

| | |
|---|---|
| ***W*** | $W$ |
| ***COPULA*** | $COPULA$ |
| ***MPROP*** | $MPROP$ |

*Binder*: $\forall_{\boldsymbol{a}}$ $\exists_{\boldsymbol{a}}$
*Right Infix 100*:
$\Vdash$ $\models$
*Right Infix 200*:
$\Leftrightarrow_{\boldsymbol{a}}$
*Right Infix 210*:
$\Rightarrow_{\boldsymbol{a}}$
*Right Infix 220*:
$\wedge_{\boldsymbol{a}}$
*Postfix 100*: ***not***

***actual_world***
***extension*** $\vdash T$
***not*** $\vdash \forall\ pred\bullet\ (pred\ not) = (\lambda\ pa\ t\ w\bullet \neg\ pred\ pa\ t\ w)$
***All*** $\vdash \forall\ s\ r\ p$
$\bullet\ All\ s\ r\ p$
$= (\lambda\ w\bullet \forall\ z\bullet\ z \in extension\ s \wedge z \in w \Rightarrow r\ z\ p\ w)$
***Some*** $\vdash \forall\ s\ r\ p$
$\bullet\ Some\ s\ r\ p$
$= (\lambda\ w\bullet \exists\ z\bullet\ z \in extension\ s \wedge z \in w \wedge r\ z\ p\ w)$
***are*** $\vdash \forall\ i\ t\bullet\ are\ i\ t = (\lambda\ w\bullet\ i \in extension\ t)$
$\Diamond$ $\vdash \forall\ p\bullet \Diamond\ p = (\lambda\ w\bullet \exists\ w'\bullet\ p\ w')$
$\Box$ $\vdash \forall\ p\bullet \Box\ p = (\lambda\ w\bullet \forall\ w'\bullet\ p\ w')$
$\neg_{\boldsymbol{a}}$ $\vdash \forall\ p\bullet (\neg\ p) = (\lambda\ w\bullet \neg\ p\ w)$
$\wedge_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet (p \wedge q) = (\lambda\ w\bullet\ p\ w \wedge q\ w)$
$\Rightarrow_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet (p \Rightarrow q) = (\lambda\ w\bullet\ p\ w \Rightarrow q\ w)$
$\Leftrightarrow_{\boldsymbol{a}}$ $\vdash \forall\ p\ q\bullet (p \Leftrightarrow q) = (\lambda\ w\bullet\ p\ w \Leftrightarrow q\ w)$
$\forall_{\boldsymbol{a}}$ $\vdash \forall\ mpf\bullet \$\forall\ mpf = (\lambda\ w\bullet \forall\ t\bullet\ mpf\ t\ w)$
$\exists_{\boldsymbol{a}}$ $\vdash \forall\ mpf\bullet \$\exists\ mpf = (\lambda\ w\bullet \exists\ t\bullet\ mpf\ t\ w)$
$\models$ $\vdash \forall\ lp\ c$
$\bullet\ lp \models c$
$\Leftrightarrow Fold\ (\lambda\ p\ t\bullet\ p\ actual_world \wedge t)\ lp\ T$
$\Rightarrow c\ actual_world$
$\Vdash$ $\vdash \forall\ lp\ c$
$\bullet\ lp \Vdash c \Leftrightarrow (\forall\ w\bullet Fold\ (\lambda\ p\ t\bullet\ p\ w \wedge t)\ lp\ T \Rightarrow c\ w)$
***individual*** $\vdash \forall\ A\bullet\ individual\ A = (\lambda\ w\bullet \exists\ a\bullet\ extension\ A = \{a\})$

***are_not_lemma***
$\vdash All\ B\ (are\ not)\ A = All\ A\ (are\ not)\ B$
***some_are_lemma***
$\vdash Some\ B\ are\ A = Some\ A\ are\ B$
***are_conv1*** $\vdash [All\ B\ (are\ not)\ A] \Vdash All\ A\ (are\ not)\ B$
***are_conv3*** $\vdash [Some\ B\ are\ A] \Vdash Some\ A\ are\ B$

***◇_conv*** $\vdash [P] \Vdash Q \Rightarrow [\Diamond\ P] \Vdash \Diamond\ Q$

***□_conv*** $\vdash [P] \Vdash Q \Rightarrow [\Box\ P] \Vdash \Box\ Q$

***◇AllBareA_thm***

$\vdash [] \Vdash \Diamond\ (All\ B\ are\ A)$

***◇AarenotA_thm***

$\vdash [] \vDash \Diamond\ (All\ A\ (are\ not)\ A)$

***□_elim_thm*** $\vdash [\Box\ P] \vDash P$

***□_intro_thm*** $\vdash [P] \vDash \Diamond\ P$

***□_◇_thm*** $\vdash [\Box\ P] \vDash \Diamond\ P$

***¬_All_conv_thm***

$\vdash (\neg\ All\ A\ cop\ B) = Some\ A\ (cop\ not)\ B$

***¬_All_not_conv_thm2***

$\vdash (\neg\ All\ A\ (cop\ not)\ B) = Some\ A\ cop\ B$

***¬_Some_conv_thm***

$\vdash (\neg\ Some\ A\ cop\ B) = All\ A\ (cop\ not)\ B$

***¬_Some_not_conv_thm***

$\vdash (\neg\ Some\ A\ (cop\ not)\ B) = All\ A\ cop\ B$

***Barbara_are*** $\vdash [All\ M\ are\ P;\ All\ \mathbb{S}\ are\ M] \Vdash All\ \mathbb{S}\ are\ P$

***Celarent_are*** $\vdash [All\ M\ (are\ not)\ P;\ All\ \mathbb{S}\ are\ M] \Vdash All\ \mathbb{S}\ (are\ not)\ P$

***Darii_are*** $\vdash [All\ M\ are\ P;\ Some\ \mathbb{S}\ are\ M] \Vdash Some\ \mathbb{S}\ are\ P$

***Ferio_are*** $\vdash [All\ M\ (are\ not)\ P;\ Some\ \mathbb{S}\ are\ M]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

***Cesare_are*** $\vdash [All\ P\ (are\ not)\ M;\ All\ \mathbb{S}\ are\ M] \Vdash All\ \mathbb{S}\ (are\ not)\ P$

***Camestres_are***

$\vdash [All\ P\ are\ M;\ All\ \mathbb{S}\ (are\ not)\ M] \Vdash All\ \mathbb{S}\ (are\ not)\ P$

***Festino_are*** $\vdash [All\ P\ (are\ not)\ M;\ Some\ \mathbb{S}\ are\ M]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

***Baroco_are*** $\vdash [All\ P\ are\ M;\ Some\ \mathbb{S}\ (are\ not)\ M]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

***Disamis_are*** $\vdash [Some\ M\ are\ P;\ All\ M\ are\ \mathbb{S}] \Vdash Some\ \mathbb{S}\ are\ P$

***Datisi_are*** $\vdash [All\ M\ are\ P;\ Some\ M\ are\ \mathbb{S}] \Vdash Some\ \mathbb{S}\ are\ P$

***Bocardo_are*** $\vdash [Some\ M\ (are\ not)\ P;\ All\ M\ are\ \mathbb{S}]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

***Ferison_are*** $\vdash [All\ M\ (are\ not)\ P;\ Some\ M\ are\ \mathbb{S}]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

***Camenes_are*** $\vdash [All\ P\ are\ M;\ All\ M\ (are\ not)\ \mathbb{S}] \Vdash All\ \mathbb{S}\ (are\ not)\ P$

***Dimaris_are*** $\vdash [Some\ P\ are\ M;\ All\ M\ are\ \mathbb{S}] \Vdash Some\ \mathbb{S}\ are\ P$

***Fresison_are*** $\vdash [All\ P\ (are\ not)\ M;\ Some\ M\ are\ \mathbb{S}]$
$\Vdash Some\ \mathbb{S}\ (are\ not)\ P$

# Chapter 4

# Russell and Wittgenstein

## 4.1 The Theory tract01

*rbjmisc*

| | |
|---|---|
| $\boldsymbol{N}$ | $BOOL\ WORLD \rightarrow BOOL$ |
| $\boldsymbol{M}$ | $('a \rightarrow BOOL) \rightarrow BOOL$ |
| $\boldsymbol{N_2}$ | $BOOL\ WORLD \rightarrow BOOL$ |

***OBJECT***
***NAME***

| | |
|---|---|
| $'a$ ***WORLD*** | $'a\ WORLD$ |
| $('a, 'b, 'c)$ ***CONSTRUCTION*** | $('a, 'b, 'c)\ CONSTRUCTION$ |

| | |
|---|---|
| $\boldsymbol{N}$ | $\vdash \forall\ s\bullet\ N\ s \Leftrightarrow (\forall\ p\bullet\ p \in s \Rightarrow \neg\ p)$ |
| $\boldsymbol{M}$ | $\vdash \forall\ f\bullet\ M\ f \Leftrightarrow (\forall\ x\bullet \neg\ f\ x)$ |
| $\boldsymbol{N_2}$ | $\vdash ConstSpec\ (\lambda\ N_2'\bullet \forall\ s\bullet\ N\ s \Leftrightarrow (\forall\ p\bullet\ p \in s \Rightarrow \neg\ p))\ N_2$ |

| | |
|---|---|
| $\boldsymbol{N_\neg_thm}$ | $\vdash \forall\ p\bullet \neg\ p \Leftrightarrow N\ \{p\}$ |
| $\boldsymbol{N_\wedge_thm}$ | $\vdash \forall\ p\ q\bullet\ p \wedge q \Leftrightarrow N\ \{N\ \{p\};\ N\ \{q\}\}$ |
| $\boldsymbol{N_\forall_thm}$ | $\vdash \forall\ pf\bullet\ \$\forall\ pf \Leftrightarrow N\ \{p \mid \exists\ x\bullet\ p \Leftrightarrow N\ \{pf\ x\}\}$ |
| $\boldsymbol{M_\neg_thm}$ | $\vdash \forall\ p\bullet \neg\ p \Leftrightarrow M\ (\lambda\ x\bullet\ p)$ |
| $\boldsymbol{M_\wedge_thm}$ | $\vdash \forall\ p\ q$ |
| | $\bullet\ p \wedge q$ |
| | $\Leftrightarrow M\ (\lambda\ x\bullet\ if\ x\ then\ M\ (\lambda\ x\bullet\ p)\ else\ M\ (\lambda\ x\bullet\ q))$ |
| $\boldsymbol{M_\forall_thm}$ | $\vdash \forall\ pf\bullet\ \$\forall\ pf \Leftrightarrow M\ (\lambda\ x\bullet \neg\ pf\ x)$ |
| $\boldsymbol{M_\exists_thm}$ | $\vdash \forall\ pf\bullet\ \$\exists\ pf \Leftrightarrow M\ (\lambda\ x\bullet\ M\ (\lambda\ x\bullet\ pf\ x))$ |

# Chapter 5

# Grice

## 5.1 The Theory grice

*rbjmisc*

*griceS* *griceC*

| | |
|---|---|
| $\$\models$ | $BOOL\ LIST \to BOOL \to BOOL$ |
| $\$\boldsymbol{committal}$ | $('a \to BOOL) \to ('a \to BOOL) \to BOOL$ |

*Right Infix 4*: $\models$
*Right Infix 100*:
***committal***

| | |
|---|---|
| $\models$ | $\vdash \forall\ al\ c\bullet\ (al \models c) \Leftrightarrow \forall_L\ al \Rightarrow c$ |
| ***committal*** | $\vdash \forall\ P\ \phi\bullet\ P\ committal\ \phi \Leftrightarrow (\forall\ \alpha\bullet\ \phi\ \alpha \Rightarrow P\ \alpha)$ |

| | |
|---|---|
| ***ASS*** | $\vdash \forall\ \phi\bullet\ [\phi] \models \phi$ |
| ***RAA*** | $\vdash \forall\ \phi\ \psi\ \zeta\ \Gamma\bullet\ ([\phi]\ @\ \Gamma \models \psi \wedge \neg\ \psi) \Rightarrow (\Gamma \models \neg\ \phi)$ |
| ***DN*** | $\vdash \forall\ \phi\bullet\ [\neg\ \neg\ \phi] \models \phi$ |
| $\wedge\boldsymbol{I}$ | $\vdash \forall\ \phi\ \psi\bullet\ [\phi;\ \psi] \models \phi \wedge \psi$ |
| $\wedge\boldsymbol{E}$ | $\vdash \forall\ \phi\ \psi\bullet\ ([\phi \wedge \psi] \models \phi) \wedge ([\phi \wedge \psi] \models \psi)$ |
| $\vee\boldsymbol{I}$ | $\vdash \forall\ \phi\ \psi\bullet\ ([\phi] \models \phi \vee \psi) \wedge ([\phi] \models \psi \vee \phi)$ |
| $\vee\boldsymbol{E}$ | $\vdash \forall\ \phi\ \psi\ \zeta\ \Gamma\ \Delta\ \Theta$ |
| | $\bullet\ ([\phi]\ @\ \Gamma \models \zeta) \wedge ([\psi]\ @\ \Delta \models \zeta) \wedge (\Theta \models \phi \vee \psi)$ |
| | $\Rightarrow (\Gamma\ @\ \Delta\ @\ \Theta \models \zeta)$ |
| ***CP*** | $\vdash \forall\ \phi\ \psi\ \Gamma\bullet\ ([\phi]\ @\ \Gamma \models \psi) \Rightarrow (\Gamma \models \phi \Rightarrow \psi)$ |
| ***MPP*** | $\vdash \forall\ \phi\ \psi\bullet\ [\phi \Rightarrow \psi;\ \phi] \models \psi$ |
| $\forall\boldsymbol{I}$ | $\vdash \forall\ \Gamma\ P\bullet\ (\forall\ x\bullet\ \Gamma \models P\ x) \Rightarrow (\Gamma \models (\forall\ x\bullet\ P\ x))$ |
| $\forall\boldsymbol{E}$ | $\vdash \forall\ \Gamma\ P\ c\bullet\ (\Gamma \models (\forall\ x\bullet\ P\ x)) \Rightarrow (\Gamma \models P\ c)$ |
| $\exists\boldsymbol{I}$ | $\vdash \forall\ \Gamma\ P\ x\bullet\ (\Gamma \models P\ x) \Rightarrow (\Gamma \models (\exists\ x\bullet\ P\ x))$ |
| $\exists\boldsymbol{E}$ | $\vdash \forall\ P\ Q\bullet\ (\forall\ x\bullet\ [P\ x] \models Q) \Rightarrow ([\exists\ x\bullet\ P\ x] \models Q)$ |
| ***P_2*** | $\vdash \forall\ P\ \phi\bullet\ P\ committal\ \phi \Rightarrow P\ committal\ (\lambda\ \alpha\bullet\ \neg\ \neg\ \phi\ \alpha)$ |
| ***P_3*** | $\vdash \forall\ P\ \phi\ \psi$ |
| | $\bullet\ P\ committal\ \phi \vee P\ committal\ \psi$ |
| | $\Rightarrow P\ committal\ (\lambda\ \alpha\bullet\ \phi\ \alpha \wedge \psi\ \alpha)$ |
| ***P_4*** | $\vdash \forall\ P\ \phi\ \psi$ |
| | $\bullet\ P\ committal\ \phi \wedge P\ committal\ \psi$ |
| | $\Rightarrow P\ committal\ (\lambda\ \alpha\bullet\ \phi\ \alpha \vee \psi\ \alpha)$ |
| ***P_5*** | $\vdash \forall\ P\ \phi\ \psi$ |
| | $\bullet\ P\ committal\ (\lambda\ \alpha\bullet\ \neg\ \phi\ \alpha) \wedge P\ committal\ \psi$ |

$\Rightarrow P\ \mathit{committal}\ (\lambda\ \alpha\bullet\ \phi\ \alpha \Rightarrow \psi\ \alpha)$

**$P$_6$\forall$** $\vdash \forall\ P\ \phi$

$\bullet\ (\exists\ \beta\bullet\ P\ \mathit{committal}\ \phi\ \beta)$

$\Rightarrow P\ \mathit{committal}\ (\lambda\ \alpha\bullet\ \forall\ \beta\bullet\ \phi\ \beta\ \alpha)$

**$P$_6$\exists b$** $\vdash \forall\ P\ \phi$

$\bullet\ (\forall\ \beta\bullet\ P\ \mathit{committal}\ \phi\ \beta)$

$\Rightarrow P\ \mathit{committal}\ (\lambda\ \alpha\bullet\ \exists\ \beta\bullet\ \phi\ \beta\ \alpha)$

# Chapter 6

# Logical Foundations

# Chapter 7

# Universal Algebra and Lattice Theory

## 7.1 The Theory unalg

*equiv*

*uacat latt2 lattice*

| | |
|---|---|
| ***SOps*** | $'a\ STRUCT \to (STRING, 'a\ UOP)\ IX$ |
| ***SCar*** | $'a\ STRUCT \to 'a\ \mathbb{P}$ |
| ***MkSTRUCT*** | $'a\ \mathbb{P} \to (STRING, 'a\ UOP)\ IX \to 'a\ STRUCT$ |
| ***Arity$_u$*** | $'a\ STRUCT \to STRING \to \mathbb{N}$ |
| ***Oper*** | $'a\ STRUCT \to STRING \to (\mathbb{N} \to 'a) \to 'a$ |
| ***Sig*** | $'a\ STRUCT \to SIG$ |
| ***Arity$_i$*** | $SIG \to STRING \to \mathbb{N}$ |
| ***StructInc*** | $'a\ STRUCT \to 'a\ STRUCT \to BOOL$ |
| ***ClosedOp*** | $'a\ \mathbb{P} \to 'a\ UOP \to BOOL$ |
| ***ClosedStruct*** | $'a\ STRUCT \to BOOL$ |
| ***pack0op*** | $'a \to 'a\ UOP$ |
| ***unpack0op*** | $((\mathbb{N} \to 'a) \to 'a) \to 'a$ |
| ***pack1op*** | $('a \to 'a) \to 'a\ UOP$ |
| ***unpack1op*** | $((\mathbb{N} \to 'a) \to 'a) \to 'a \to 'a$ |
| ***pack2op*** | $('a \to 'a \to 'a) \to 'a\ UOP$ |
| ***unpack2op*** | $((\mathbb{N} \to 'a) \to 'a) \to 'a \to 'a \to 'a$ |
| ***FunClosed*** | $'b\ STRUCT \times ('b \to 'c) \times 'c\ STRUCT \to BOOL$ |
| ***OpRespect*** | $'b\ \mathbb{P} \times ('b \to 'c) \times \mathbb{N}$ |
| | $\to ((\mathbb{N} \to 'b) \to 'b)$ |
| | $\to ((\mathbb{N} \to 'c) \to 'c)$ |
| | $\to BOOL$ |
| ***HomOp*** | $'b\ STRUCT \times ('b \to 'c) \times 'c\ STRUCT \to STRING \to BOOL$ |
| ***HomOps*** | $'b\ STRUCT \times ('b \to 'c) \times 'c\ STRUCT \to BOOL$ |
| ***AlgHom*** | $'b\ STRUCT \times ('b \to 'c) \times 'c\ STRUCT \to BOOL$ |
| ***QuotientAlg*** | |
| | $'b\ STRUCT \to ('b \to 'b \to BOOL) \to 'b\ \mathbb{P}\ STRUCT$ |
| ***VExpr*** | $\mathbb{N} \to 'a\ EXPR$ |
| ***VExprs*** | $\mathbb{N} \to 'a\ EXPR\ \mathbb{P}$ |
| ***CExprs*** | $SIG \to 'a\ EXPR\ \mathbb{P} \to 'a\ EXPR\ \mathbb{P}$ |
| ***ExprClosed*** | $SIG \to 'a\ EXPR\ \mathbb{P} \to BOOL$ |
| ***Exprs*** | $SIG \to \mathbb{N} \to 'a\ EXPR\ \mathbb{P}$ |

| | |
|---|---|
| $\sqsubseteq$ | $StructInc : 'a\ STRUCT \to 'a\ STRUCT \to BOOL$ |
| $/$ | $QuotientAlg$ |
| | $: 'a\ STRUCT \to ('a \to 'a \to BOOL) \to 'a\ \mathbb{P}\ STRUCT$ |

## $'1$ ***STRUCT***

| | |
|---|---|
| ***SIG*** | *SIG* |
| $'a$ ***EXPR*** | $'a$ *EXPR* |

*Right Infix 200*:

$*_c \quad *_d$

| | |
|---|---|
| ***STRUCT*** | $\vdash \exists\, f\bullet\ TypeDefn\ (\lambda\ x\bullet\ T)\ f$ |
| ***MkSTRUCT*** | |
| ***SCar*** | |
| ***SOps*** | $\vdash \forall\ t\ x1\ x2$ |
| | $\bullet\ SCar\ (MkSTRUCT\ x1\ x2) = x1$ |
| | $\wedge\ SOps\ (MkSTRUCT\ x1\ x2) = x2$ |
| | $\wedge\ MkSTRUCT\ (SCar\ t)\ (SOps\ t) = t$ |
| $\boldsymbol{Arity_u}$ | $\vdash \forall\ A\ n\bullet\ Arity_u\ A\ n = Fst\ (ValueOf\ (SOps\ A\ n))$ |
| ***Oper*** | $\vdash \forall\ A\ n\bullet\ Oper\ A\ n = Snd\ (ValueOf\ (SOps\ A\ n))$ |
| ***Sig*** | $\vdash \forall\ s\bullet\ Sig\ s = IxCompIx\ (SOps\ s)\ (\lambda\ x\bullet\ Value\ (Fst\ x))$ |
| $\boldsymbol{Arity_i}$ | $\vdash \forall\ A\ n\bullet\ Arity_i\ A\ n = ValueOf\ (A\ n)$ |
| ***StructInc*** | $\vdash \forall\ s\ t\bullet\ s \sqsubseteq t \Leftrightarrow SCar\ s = SCar\ t \wedge SOps\ s \sqsubseteq SOps\ t$ |
| ***ClosedOp*** | $\vdash \forall\ s\ p$ |
| | $\bullet\ ClosedOp\ s\ p$ |
| | $\Leftrightarrow (\forall\ f\bullet\ (\forall\ i\bullet\ i < Fst\ p \Rightarrow f\ i \in s) \Rightarrow Snd\ p\ f \in s)$ |
| ***ClosedStruct*** | $\vdash \forall\ s$ |
| | $\bullet\ ClosedStruct\ s$ |
| | $\Leftrightarrow (\forall\ p\bullet\ p \in IxRan\ (SOps\ s) \Rightarrow ClosedOp\ (SCar\ s)\ p)$ |
| ***pack0op*** | $\vdash \forall\ c\bullet\ pack0op\ c = (0,\ (\lambda\ is\bullet\ c))$ |
| ***unpack0op*** | $\vdash \forall\ f\bullet\ unpack0op\ f = f\ (\epsilon\ x\bullet\ T)$ |
| ***pack1op*** | $\vdash \forall\ f\bullet\ pack1op\ f = (1,\ (\lambda\ is\bullet\ f\ (is\ 0)))$ |
| ***unpack1op*** | $\vdash \forall\ f\bullet\ unpack1op\ f = (\lambda\ x\bullet\ f\ (\lambda\ y\bullet\ x))$ |
| ***pack2op*** | $\vdash \forall\ f\bullet\ pack2op\ f = (2,\ (\lambda\ is\bullet\ f\ (is\ 0)\ (is\ 1)))$ |
| ***unpack2op*** | $\vdash \forall\ f$ |
| | $\bullet\ unpack2op\ f$ |
| | $= (\lambda\ x\ y\bullet\ f\ (\lambda\ z\bullet\ if\ z = 0\ then\ x\ else\ y))$ |
| ***FunClosed*** | $\vdash \forall\ A\ f\ B$ |
| | $\bullet\ FunClosed\ (A,\ f,\ B)$ |
| | $\Leftrightarrow (\forall\ x\bullet\ x \in SCar\ A \Rightarrow f\ x \in SCar\ B)$ |
| ***OpRespect*** | $\vdash \forall\ D\ f\ n\ op1\ op2$ |
| | $\bullet\ OpRespect\ (D,\ f,\ n)\ op1\ op2$ |
| | $\Leftrightarrow (\forall\ g$ |
| | $\bullet\ FunImage\ g\ \{i \mid i < n\} \subseteq D$ |
| | $\Rightarrow f\ (op1\ g) = op2\ (\lambda\ x\bullet\ f\ (g\ x)))$ |

***HomOp***

$$
\begin{array}{l}
\vdash \forall\ A\ s\ B\ f \\
\quad \bullet\ HomOp\ (A,\ f,\ B)\ s \\
\qquad \Leftrightarrow\ OpRespect \\
\qquad\quad (SCar\ A,\ f,\ Arity_u\ A\ s) \\
\qquad\quad (Oper\ A\ s) \\
\qquad\quad (Oper\ B\ s)
\end{array}
$$

***HomOps***

$$
\begin{array}{l}
\vdash \forall\ A\ B\ f \\
\quad \bullet\ HomOps\ (A,\ f,\ B) \\
\qquad \Leftrightarrow\ (\forall\ s \bullet\ s \in IxDom\ (SOps\ A) \Rightarrow HomOp\ (A,\ f,\ B)\ s)
\end{array}
$$

***AlgHom***

$$
\begin{array}{l}
\vdash \forall\ A\ f\ B \\
\quad \bullet\ AlgHom\ (A,\ f,\ B) \\
\qquad \Leftrightarrow\ Sig\ A \sqsubseteq Sig\ B \\
\qquad\quad \wedge\ FunClosed\ (A,\ f,\ B) \\
\qquad\quad \wedge\ HomOps\ (A,\ f,\ B)
\end{array}
$$

***QuotientAlg***

$$
\begin{array}{l}
\vdash \forall\ A\ r \\
\quad \bullet\ A\ /\ r \\
\qquad = MkSTRUCT \\
\qquad\quad (SCar\ A\ /\ r) \\
\qquad\quad (IxComp\ (SOps\ A)\ (UniOpLift\ (SCar\ A,\ r)))
\end{array}
$$

***VExpr***

$$\vdash \forall\ n \bullet\ VExpr\ n = (\lambda\ a\ va \bullet\ va\ n)$$

***VExprs***

$$\vdash \forall\ n \bullet\ VExprs\ n = \{p | \exists\ m \bullet\ m < n \wedge p = VExpr\ m\}$$

***CExprs***

$$
\begin{array}{l}
\vdash \forall\ sig\ es \\
\quad \bullet\ CExprs\ sig\ es \\
\qquad = \{e \\
\qquad | \exists\ name\ arity\ am \\
\qquad\quad \bullet\ name \in IxDom\ sig \\
\qquad\qquad \wedge\ sig\ name = Value\ arity \\
\qquad\qquad \wedge\ (\forall\ i \bullet\ i < arity \Rightarrow am\ i \in es) \\
\qquad\qquad \wedge\ e \\
\qquad\qquad\quad = (\lambda\ struct\ va \\
\qquad\qquad\quad \bullet\ Snd \\
\qquad\qquad\qquad (ValueOf\ (SOps\ struct\ name)) \\
\qquad\qquad\qquad (\lambda\ i \bullet\ am\ i\ struct\ va))\}
\end{array}
$$

***ExprClosed***

$$\vdash \forall\ s\ es \bullet\ ExprClosed\ s\ es \Leftrightarrow CExprs\ s\ es \subseteq es$$

***Exprs***

$$
\begin{array}{l}
\vdash \forall\ s\ n \\
\quad \bullet\ Exprs\ s\ n = \bigcap \{ps | ExprClosed\ s\ ps \wedge VExprs\ n \subseteq ps\}
\end{array}
$$

***$Arity_u$_lemma***

$$
\begin{array}{l}
\vdash \forall\ d\ l\ n \\
\quad \bullet\ Arity_u\ (MkSTRUCT\ d\ (IxPack\ l))\ n \\
\qquad = Fst\ (ValueOf\ (IxPack\ l\ n))
\end{array}
$$

***Oper_lemma***

$$\vdash \forall\ d\ l\ n$$

$\bullet\ Oper\ (MkSTRUCT\ d\ (IxPack\ l))\ n$
$\quad = Snd\ (ValueOf\ (IxPack\ l\ n))$

***IxDom_Sig_thm***

$\vdash \forall\ S \bullet\ IxDom\ (Sig\ S) = IxDom\ (SOps\ S)$

***∈_IxDom_Sig_thm***

$\vdash \forall\ S\ x \bullet\ x \in IxDom\ (Sig\ S) \Leftrightarrow x \in IxDom\ (SOps\ S)$

***sig_arity_lemma***

$\vdash \forall\ A\ n$
$\bullet\ n \in IxDom\ (Sig\ A) \Rightarrow Arity_i\ (Sig\ A)\ n = Arity_u\ A\ n$

***IxDom_Sig_SOps_thm***

$\vdash \forall\ A \bullet\ IxDom\ (Sig\ A) = IxDom\ (SOps\ A)$

***SigInc_IxDom_⊆_thm***

$\vdash \forall\ A\ B \bullet\ Sig\ A \sqsubseteq Sig\ B \Rightarrow IxDom\ (Sig\ A) \subseteq IxDom\ (Sig\ B)$

***SigInc_IxDom_Sops_⊆_thm***

$\vdash \forall\ A\ B$
$\bullet\ Sig\ A \sqsubseteq Sig\ B \Rightarrow IxDom\ (SOps\ A) \subseteq IxDom\ (SOps\ B)$

***SigInc_Arity$_i$_thm***

$\vdash \forall\ A\ B\ n$
$\bullet\ Sig\ A \sqsubseteq Sig\ B \wedge n \in IxDom\ (Sig\ A)$
$\quad \Rightarrow Arity_i\ (Sig\ A)\ n = Arity_i\ (Sig\ B)\ n$

***SigInc_Arity$_u$_thm***

$\vdash \forall\ A\ B\ n$
$\bullet\ Sig\ A \sqsubseteq Sig\ B \wedge n \in IxDom\ (Sig\ A)$
$\quad \Rightarrow Arity_u\ A\ n = Arity_u\ B\ n$

***fst_packop_lemma***

$\vdash (\forall\ k \bullet\ Fst\ (pack0op\ k) = 0)$
$\quad \wedge (\forall\ k \bullet\ Fst\ (pack1op\ k) = 1)$
$\quad \wedge (\forall\ k \bullet\ Fst\ (pack2op\ k) = 2)$

***UniOpLift_pack0op_lemma***

$\vdash \forall\ C\ r\ op$
$\bullet\ UniOpLift\ (C,\ r)\ (pack0op\ op)$
$\quad = pack0op\ (EquivClass\ (C,\ r)\ op)$

***UniOpLift_pack1op_lemma***

$\vdash \forall\ C\ r\ op$
$\bullet\ UniOpLift\ (C,\ r)\ (pack1op\ op)$
$\quad = pack1op\ (op\ MonOpLift\ (C,\ r))$

***UniOpLift_pack2op_lemma***

$\vdash \forall\ C\ r\ op$
$\bullet\ UniOpLift\ (C,\ r)\ (pack2op\ op)$
$\quad = pack2op\ (op\ DyOpLift\ (C,\ r))$

***FunClosed_trans_thm***

$\vdash \forall\ A\ f\ B\ g\ C$
$\bullet\ FunClosed\ (A,\ f,\ B) \wedge FunClosed\ (B,\ g,\ C)$

$\Rightarrow$ *FunClosed* $(A,\ g\ o\ f,\ C)$

***FunClosed_FunImage_thm***

$\vdash \forall\ A\ f\ B$
$\bullet$ *FunClosed* $(A,\ f,\ B) \Leftrightarrow$ *FunImage* $f$ $(SCar\ A) \subseteq SCar\ B$

***OpRespect_pack0op_lemma***

$\vdash \forall\ D\ f\ d\ c$
$\bullet$ *OpRespect*
$(D,\ f,\ 0)$
$(Snd\ (pack0op\ d))$
$(Snd\ (pack0op\ c))$
$\Leftrightarrow f\ d = c$

***OpRespect_pack1op_lemma***

$\vdash \forall\ D\ f\ d\ c$
$\bullet$ *OpRespect*
$(D,\ f,\ 1)$
$(Snd\ (pack1op\ d))$
$(Snd\ (pack1op\ c))$
$\Leftrightarrow (\forall\ x \bullet\ x \in D \Rightarrow f\ (d\ x) = c\ (f\ x))$

***OpRespect_pack2op_lemma***

$\vdash \forall\ D\ f\ \$*_d\ \$*_c$
$\bullet$ *OpRespect*
$(D,\ f,\ 2)$
$(Snd\ (pack2op\ \$*_d))$
$(Snd\ (pack2op\ \$*_c))$
$\Leftrightarrow (\forall\ x\ y$
$\bullet\ x \in D \wedge y \in D \Rightarrow f\ (x *_d y) = f\ x *_c f\ y)$

***HomOps_o_thm*** $\vdash \forall\ A\ f\ B\ g\ C$
$\bullet$ *HomOps* $(A,\ f,\ B)$
$\wedge$ *Sig* $A \sqsubseteq$ *Sig* $B$
$\wedge$ *FunClosed* $(A,\ f,\ B)$
$\wedge$ *HomOps* $(B,\ g,\ C)$
$\Rightarrow$ *HomOps* $(A,\ g\ o\ f,\ C)$

***AlgHom_o_thm*** $\vdash \forall\ A\ f\ B\ g\ C$
$\bullet$ *AlgHom* $(A,\ f,\ B) \wedge$ *AlgHom* $(B,\ g,\ C)$
$\Rightarrow$ *AlgHom* $(A,\ g\ o\ f,\ C)$

***QuotientAlg_Sig_lemma***

$\vdash \forall\ A\ r \bullet$ *Sig* $A \sqsubseteq$ *Sig* $(A\ /\ r)$

***FunClosed_EquivClass_lemma***

$\vdash \forall\ A\ r \bullet$ *FunClosed* $(A,$ *EquivClass* $(SCar\ A,\ r),\ A\ /\ r)$

***AlgHom_EquivClass_HomOps_lemma***

$\vdash \forall\ A\ r$
$\bullet$ *AlgHom* $(A,$ *EquivClass* $(SCar\ A,\ r),\ A\ /\ r)$
$\Leftrightarrow$ *HomOps* $(A,$ *EquivClass* $(SCar\ A,\ r),\ A\ /\ r)$

## 7.2 The Theory lattice

*unalg*

$\boldsymbol{Meet_L}$ $'a\ LAT \to 'a \to 'a \to 'a$
$\boldsymbol{Join_L}$ $'a\ LAT \to 'a \to 'a \to 'a$
$\boldsymbol{Car_L}$ $'a\ LAT \to 'a\ \mathbb{P}$
$\boldsymbol{MkLAT}$ $'a\ \mathbb{P} \to ('a \to 'a \to 'a) \to ('a \to 'a \to 'a) \to 'a\ LAT$
$\boldsymbol{IsLattice}$ $'a\ LAT \to BOOL$
$\boldsymbol{QuotientLattice}$
$'a\ LAT \to ('a \to 'a \to BOOL) \to 'a\ \mathbb{P}\ LAT$
$\boldsymbol{LeLat}$ $'a\ LAT \to 'a \to 'a \to BOOL$

/ *QuotientLattice*
$: 'a\ LAT \to ('a \to 'a \to BOOL) \to 'a\ \mathbb{P}\ LAT$

$'1\ \boldsymbol{LAT}$

*Right Infix 210*:
$\leq_{\boldsymbol{L}}$ $\leq_{\boldsymbol{M}}$
*Right Infix 235*:
$\vee_{\boldsymbol{L}}$ $\vee_{\boldsymbol{M}}$
*Right Infix 240*:
$\wedge_{\boldsymbol{L}}$ $\wedge_{\boldsymbol{M}}$

$\boldsymbol{LAT}$ $\vdash \exists\ f \bullet\ TypeDefn\ (\lambda\ x \bullet\ T)\ f$
$\boldsymbol{MkLAT}$
$\boldsymbol{Car_L}$
$\boldsymbol{Join_L}$
$\boldsymbol{Meet_L}$ $\vdash \forall\ t\ x1\ x2\ x3$
$\bullet\ Car_L\ (MkLAT\ x1\ x2\ x3) = x1$
$\wedge\ Join_L\ (MkLAT\ x1\ x2\ x3) = x2$
$\wedge\ Meet_L\ (MkLAT\ x1\ x2\ x3) = x3$
$\wedge\ MkLAT\ (Car_L\ t)\ (Join_L\ t)\ (Meet_L\ t) = t$
$\boldsymbol{IsLattice}$ $\vdash \forall\ L$
$\bullet\ IsLattice\ L$
$\Leftrightarrow (\forall\ C\ \$\vee_L\ \$\wedge_L$
$\bullet\ MkLAT\ C\ \$\vee_L\ \$\wedge_L = L$
$\Rightarrow (\forall\ x\ y$
$\bullet\ x \in C \wedge y \in C$

$$
\begin{array}{l}
\Rightarrow x \vee_L y \in C \\
\quad \wedge\ x \wedge_L y \in C \\
\quad \wedge\ x \vee_L y = y \vee_L x \\
\quad \wedge\ x \wedge_L y = y \wedge_L x \\
\quad \wedge\ x \wedge_L (x \vee_L y) = x \\
\quad \wedge\ x \vee_L x \wedge_L y = x \\
\quad \wedge\ (\forall\ z \\
\quad \bullet\ z \in C \\
\qquad \Rightarrow (x \vee_L y) \vee_L z = x \vee_L y \vee_L z \\
\qquad\ \ \wedge\ (x \wedge_L y) \wedge_L z \\
\qquad\qquad = x \wedge_L y \wedge_L z)))
\end{array}
$$

***QuotientLattice***

$$
\begin{array}{l}
\vdash \forall\ L\ \$\equiv_d \\
\quad \bullet\ L\ /\ \$\equiv_d \\
\qquad = (let\ D = (Car_L\ L,\ \$\equiv_d) \\
\qquad\quad in\ let\ \$\vee_L = Join_L\ L\ DyOpLift\ D \\
\qquad\qquad and\ \$\wedge_L = Meet_L\ L\ DyOpLift\ D \\
\qquad\qquad in\ MkLAT\ (EquivClasses\ D)\ \$\vee_L\ \$\wedge_L)
\end{array}
$$

***LeLat***

$$
\begin{array}{l}
\vdash \forall\ L \\
\quad \bullet\ LeLat\ L \\
\qquad = (\lambda\ x\ y \bullet \forall\ \$\vee_L \bullet\ \$\vee_L = Join_L\ L \Rightarrow x \vee_L y = y)
\end{array}
$$

***$\wedge_L$_idempot_thm***

$$
\begin{array}{l}
\vdash \forall\ L \\
\quad \bullet\ IsLattice\ L \\
\qquad \Rightarrow (\forall\ C\ \$\vee_L\ \$\wedge_L \\
\qquad \bullet\ MkLAT\ C\ \$\vee_L\ \$\wedge_L = L \\
\qquad\quad \Rightarrow (\forall\ x \bullet\ x \in C \Rightarrow x \wedge_L x = x))
\end{array}
$$

***$\vee_L$_idempot_thm***

$$
\begin{array}{l}
\vdash \forall\ L \\
\quad \bullet\ IsLattice\ L \\
\qquad \Rightarrow (\forall\ C\ \$\vee_L\ \$\wedge_L \\
\qquad \bullet\ MkLAT\ C\ \$\vee_L\ \$\wedge_L = L \\
\qquad\quad \Rightarrow (\forall\ x \bullet\ x \in C \Rightarrow x \vee_L x = x))
\end{array}
$$

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