

Title: HOL in HOL

Ref: DS/FMU/RBJ/150

Issue: 0.1

Date: 1989-10-05

Status: Draft

Type:

Author:

<i>Name</i>	<i>Location</i>	<i>Signature</i>	<i>Date</i>
-------------	-----------------	------------------	-------------

Roger Bishop Jones

Authorisation for Issue:

<i>Name</i>	<i>Function</i>	<i>Signature</i>	<i>Date</i>
-------------	-----------------	------------------	-------------

Abstract:

Distribution:

0 Document control

0.1 Contents list

0 Document control	1
0.1 Contents list	1
0.2 Document cross references	2
0.3 Changes history	2
0.4 Changes forecast	2
0.5 Abbreviations and notation	2
1 INTRODUCTION	2
1.1	2
1.2 Introducing the new theory	2
2 INTRODUCING THE NEW TYPE HOLSYN	3
2.1 The type definition	3
3 WELL FORMED TERMS	4
4 FREE VARIABLES	8
5 TYPE INSTANTIATION	9
5.1 Of Types	9
5.2 Of Terms	9

6	SUBSTITUTION	11
7	SEQUENTS	12
8	PROOFS	13
9	THEORIES	14
10	THE THEORY	14
11	SEMANTIC DOMAINS	14
12	ISSUES	15
12.1	Changes to the Language	15
12.2	Formalisation	15

0.2 Document cross references

0.3 Changes history

First version.

0.4 Changes forecast

Under development, highly incomplete, totally volatile.

0.5 Abbreviations and notation

HOL Higher Order Logic

1 INTRODUCTION

1.1

1.2 Introducing the new theory

We first introduce a new theory with Zermelo Fraenkel (pf137) as a parent. Definitions theorems and axioms from various ancestors are loaded and bound to ML variable names.

SML

```
|new_theory 'ppf150';;
|map new_parent ['string'; 'infra'];;
|map loadf ['infra'];;
|lemmas_on := false;;
```

2 INTRODUCING THE NEW TYPE HOLSYN

2.1 The type definition

SML

```

new_type 0 'TYPE';;
new_constant ('mk_vartype',":string → TYPE");;
new_constant ('mk_type',":string × TYPE list → TYPE");;
let TYPE_axiom = new_axiom('TYPE_axiom',"
  ∀(f0:string → *) (f1: string × TYPE list → * list → *)•
  ∃ (fn:TYPE → *)•
  (∀s• fn(mk_vartype s) = f0 s) ∧
  (∀s tl• fn(mk_type (s,tl)) = f1 (s, tl) (map fn tl))
");;

```

SML

```

new_type 0 'FTERM';;
new_constant ('mk_fvar',":string × TYPE → FTERM");;
new_constant ('mk_fconst',":string × TYPE → FTERM");;
new_constant ('mk_fcomb',":FTERM × FTERM → FTERM");;
new_constant ('mk_fabs',":string × TYPE × FTERM → FTERM");;

let FTERM_axiom = new_axiom('FTERM_axiom',"
  ∀
  (f0:string × TYPE → *)
  (f1:string × TYPE → *)
  (f2: FTERM × FTERM → * × * → *)
  (f3: string × TYPE × FTERM → * → *)•
  ∃ (fn:FTERM → *)•
  (∀s ty• fn (mk_fvar (s, ty))
    = f0 (s, ty))
  ∧ (∀s ty• fn(mk_fconst (s, ty))
    = f1 (s, ty))
  ∧ (∀f a• fn(mk_fcomb (f, a))
    = f2 (f, a) (fn f, fn a))
  ∧ (∀v ty b• fn(mk_fabs (v, ty, b))
    = f3 (v, ty, b) (fn b))
");;

```

HOL Constant

$$\text{dest_fvar}: \text{FTERM} \rightarrow \text{string} \times \text{TYPE}$$

$$\text{dest_fvar} \circ \text{mk_fvar} = I$$

HOL Constant

$$dest_fconst: FTERM \rightarrow string \times TYPE$$

$$dest_fconst \circ mk_fconst = I$$

HOL Constant

$$dest_fcomb: FTERM \rightarrow FTERM \times FTERM$$

$$dest_fcomb \circ mk_fcomb = I$$

HOL Constant

$$dest_fabs: FTERM \rightarrow string \times TYPE \times FTERM$$

$$dest_fabs \circ mk_fabs = I$$

3 WELL FORMED TERMS

HOL Constant

$$consistent_vartypes: (string \times TYPE \rightarrow bool) \rightarrow bool$$

$$consistent_vartypes \ sts =$$

$$\forall (s,t)(s',t') \bullet sts \ (s,t) \wedge sts \ (s',t') \wedge (s = s') \\ \Rightarrow (t = t')$$

HOL Constant

$$freevars_fset: FTERM \rightarrow (string \times TYPE \rightarrow bool)$$

$$\forall (s : string) (t : TYPE) (f : FTERM) \\ (a : FTERM) (v : string) (b : FTERM) \bullet \\ (freevars_fset \ (mk_fvar(s,t)) = \in x \bullet x = (s,t)) \\ \wedge (freevars_fset \ (mk_fconst(s,t)) = \in x \bullet F) \\ \wedge (freevars_fset \ (mk_fcomb(f, a)) = \\ (freevars_fset \ f) \cap (freevars_fset \ a)) \\ \wedge (freevars_fset \ (mk_fabs(v, t, b)) = \\ (freevars_fset \ b) \ \delta \in x \bullet x = (v,t))$$

HOL Constant

$$\begin{array}{l}
\text{type_of_term: } FTERM \rightarrow TYPE \rightarrow \text{bool} \\
\hline
\forall (s : \text{string}) (t : TYPE) (ty : TYPE) \\
(f : FTERM) (a : FTERM) (v : \text{string}) \\
(b : FTERM) \bullet \\
(\text{type_of_term } (\text{mk_fvar}(s,t)) \text{ ty} = (t = \text{ty})) \\
\wedge (\text{type_of_term } (\text{mk_fconst}(s,t)) \text{ ty} = (t = \text{ty})) \\
\wedge (\text{type_of_term } (\text{mk_fcomb}(f, a)) \text{ ty} = \\
\exists (t1 : TYPE) \bullet \\
(\text{type_of_term } f (\text{mk_type}(\text{'}\rightarrow\text{'}, [t1; \text{ty}]))) \\
\wedge (\text{type_of_term } a \text{ t1}) \\
\wedge ((\text{consistent_vartypes } o \text{ freevars_fset}) (\text{mk_fcomb}(f, a)))) \\
\wedge (\text{type_of_term } (\text{mk_fabs}(v, t, b)) \text{ ty} = \\
\exists (t1 : TYPE) \bullet \\
(\text{type_of_term } b \text{ t1}) \\
\wedge (\text{mk_type}(\text{'}\rightarrow\text{'}, [t; t1]) = \text{ty}) \\
\wedge (\text{consistent_vartypes } ((\text{freevars_fset } b) \cap (\in x \bullet x = (v,t))))))
\end{array}$$

For comparison, we give a constructive version of the above (omitting the checks for type consistency of free variables). The relation *type_of_term* is actually a partial function. We use the following gadgets to define partial functions.

SML

```
new_type_abbrev(' FAIL', ":one");;
```

HOL Constant

$$\begin{array}{l}
FAIL : FAIL \\
\hline
FAIL = one
\end{array}$$

We need the *dest_type* function.

HOL Constant

$$\begin{array}{l}
\text{dest_type : } TYPE \rightarrow (\text{string} \times TYPE \text{ list}) + FAIL \\
\hline
(\text{dest_type}(\text{mk_vartype } s) = \text{INR } FAIL) \\
\wedge (\text{dest_type}(\text{mk_type}(s, tl)) = \text{INL } (s, tl))
\end{array}$$

HOL Constant

```

| ctype_of_term: FTERM → TYPE + FAIL
|-----
|
|  $\forall (s : string) (t : TYPE) (ty: TYPE)$ 
|    $(f : FTERM) (a : FTERM) (v : FTERM)$ 
|    $(b : FTERM) \bullet$ 
|    $(ctype\_of\_term (mk\_fvar(s,t)) = INL t)$ 
|  $\wedge$   $(ctype\_of\_term (mk\_fconst(s,t)) = INL t)$ 
|  $\wedge$   $(ctype\_of\_term (mk\_fcomb(f, a)) =$ 
|    $let\ tf = ctype\_of\_term\ f\ in$ 
|    $let\ ta = ctype\_of\_term\ a\ in$ 
|    $if\ ((ISL\ tf) \wedge (ISL\ ta))$ 
|    $then\ ($ 
|      $let\ s\_tl = dest\_type\ (OUTL\ tf)\ in$ 
|      $if\ ((ISL\ s\_tl)$ 
|        $\wedge\ (FST\ (OUTL\ s\_tl) = ' \rightarrow ')$ 
|        $\wedge\ (length\ (SND\ (OUTL\ s\_tl)) = 2)$ 
|        $\wedge\ (HD\ (SND\ (OUTL\ s\_tl)) = (OUTL\ ta)))$ 
|      $then\ (INL\ (HD\ (TL\ (SND\ (OUTL\ s\_tl))))$ 
|      $else\ (INR\ FAIL))$ 
|    $else\ (INR\ FAIL))$ 
|  $\wedge$   $(ctype\_of\_term (mk\_abs(v, t, b)) =$ 
|    $let\ tb = (ctype\_of\_term\ b)\ in$ 
|    $if\ (ISL\ tb)$ 
|    $then\ (INL\ (mk\_type(' \rightarrow ', [t; OUTL\ tb])))$ 
|    $else\ (INR\ FAIL))$ 

```

HOL Constant

```

| is_wf_term : FTERM → bool
|-----
|
|  $\forall term:FTERM \bullet$ 
|  $is\_wf\_term\ term = \exists type:TYPE \bullet type\_of\_term\ term\ type$ 

```

SML

```

| sim_type_def ' TERM ' ("x:FTERM, is_wf_term x");
| let TERM_AXIOM = new_type_definition(' TERM ',
|   "p_or_choice is_wf_term",
|   TAC_PROOF(([],
|   "∃x:FTERM • (p_or_choice is_wf_term) x"),
|   REWRITE_TAC[EXISTS_p_or_choice]));
| let EQ_IMP thm = DISCH_ALL (EQ_MP thm (ASSUME ((lhs o concl) thm)));
| let new_type_lemmas type_axiom =
|   let [a1;a2;a3;a4;a5;a6] = prove_new_type_lemmas type_axiom
| in   LIST_CONJ [a1;a5;a3; GEN_ALL (EQ_IMP (SPEC_ALL a6));a2;a4];
| let TERM_LEMMAS = save_thm(' TERM_LEMMAS ', new_type_lemmas TERM_AXIOM);

```

HOL Constant

$$freevars_set : TERM \rightarrow (string \times TYPE \rightarrow bool)$$

$$freevars_set = freevars_fset \circ REP_TERM$$

HOL Constant

$$mk_var : string \times TYPE \rightarrow TERM$$

$$mk_var = ABS_TERM \circ mk_fvar$$

HOL Constant

$$mk_const : string \times TYPE \rightarrow TERM$$

$$mk_const = ABS_TERM \circ mk_fconst$$

HOL Constant

$$mk_comb : TERM \times TERM \rightarrow TERM$$

$$mk_comb = (ABS_TERM \circ mk_fcomb) \circ \in(x,y) \bullet (REP_TERM\ x, REP_TERM\ y)$$

HOL Constant

$$dest_var: TERM \rightarrow string \times TYPE$$

$$dest_var \circ mk_var = I$$

HOL Constant

$$mk_abs : TERM \times TERM \rightarrow TERM$$

$$mk_abs = (ABS_TERM \circ mk_fabs) \circ \in(x,y) \bullet (FST(dest_var\ x), \\ SND(dest_var\ x), \\ REP_TERM\ y)$$

HOL Constant

$$dest_const: TERM \rightarrow string \times TYPE$$

$$dest_const \circ mk_const = I$$

HOL Constant

$$dest_comb: TERM \rightarrow TERM \times TERM$$

$$dest_comb \circ mk_comb = I$$

HOL Constant

$$dest_abs: TERM \rightarrow TERM \times TERM$$

$$dest_abs \circ mk_abs = I$$

4 FREE VARIABLES

HOL Constant

$$\begin{array}{l}
\text{type_tyvars} : \text{TYPE} \rightarrow (\text{string} \rightarrow \text{bool}) \\
\hline
\wedge (\forall s:\text{string} \bullet \text{type_tyvars} (\text{mk_vartype } s) = \in x \bullet x=s) \\
\wedge (\forall s \text{ tl} \bullet \text{type_tyvars} (\text{mk_type}(s, \text{tl})) = \\
\quad \ominus \in x \bullet x \in (\text{map } \text{term_tyvars } \text{tl}))
\end{array}$$

HOL Constant

$$\begin{array}{l}
\text{term_types} : \text{TERM} \rightarrow (\text{TYPE} \rightarrow \text{bool}) \\
\hline
\wedge (\forall s \text{ t} \bullet \text{term_types} (\text{mk_var} (s, \text{t})) = \in x \bullet x=t) \\
\wedge (\forall s \text{ t} \bullet \text{term_types} (\text{mk_const} (s, \text{t})) = \in x \bullet x=t) \\
\wedge (\forall f \text{ a} \bullet \text{term_types} (\text{mk_comb} (f, \text{a})) = \\
\quad (\text{term_types } f) \cap (\text{term_types } \text{a})) \\
\wedge (\forall v \text{ b} \bullet \text{term_types} (\text{mk_abs}(v, \text{b})) = \\
\quad (\text{term_types } v) \cap (\text{term_types } \text{b}))
\end{array}$$

HOL Constant

$$\begin{array}{l}
\text{image} : (* \rightarrow **) \rightarrow (* \rightarrow \text{bool}) \rightarrow (** \rightarrow \text{bool}) \\
\hline
\forall \text{function set} \bullet \text{image function set} = \\
\quad \in x:*\bullet \exists y:*\bullet (x = f y) \wedge y \in \text{set}
\end{array}$$

HOL Constant

$$\begin{array}{l}
\text{types_tyvars} : (\text{TYPE} \rightarrow \text{bool}) \rightarrow (\text{string} \rightarrow \text{bool}) \\
\hline
\forall \text{types} \bullet \text{types_tyvars types} = \\
\quad \ominus ((\text{image}: (\text{TYPE} \rightarrow (\text{string} \rightarrow \text{bool})) \rightarrow (\text{TYPE} \rightarrow \text{bool}) \rightarrow ((\text{string} \rightarrow \text{bool}) \rightarrow \text{bool})) \\
\quad \text{type_tvars types})
\end{array}$$

HOL Constant

$$\begin{array}{l}
\text{term_tyvars} : \text{TERM} \rightarrow (\text{string} \rightarrow \text{bool}) \\
\hline
\text{term_tyvars} = \text{types_tyvars } o \text{ term_types}
\end{array}$$

HOL Constant

$$\begin{array}{l}
\text{terms_tyvars} : (\text{TERM} \rightarrow \text{bool}) \rightarrow (\text{string} \rightarrow \text{bool}) \\
\hline
\forall \text{terms}:(\text{TERM} \rightarrow \text{bool}) \bullet \\
\text{terms_tyvars terms} = \\
\quad \ominus ((\text{image}:(\text{TERM} \rightarrow (\text{string} \rightarrow \text{bool})) \rightarrow (\text{TERM} \rightarrow \text{bool}) \rightarrow ((\text{string} \rightarrow \text{bool}) \rightarrow \text{bool})) \\
\quad \text{term_tvars terms})
\end{array}$$

HOL Constant

$$l_t_s : (* list) \rightarrow (* \rightarrow bool)$$

$$l_t_s\ s\ e = e \in s$$

5 TYPE INSTANTIATION

5.1 Of Types

HOL Constant

$$inst_typef : (string \rightarrow TYPE) \rightarrow TYPE \rightarrow TYPE$$

$$\begin{aligned} & \forall (f: string \rightarrow TYPE) \bullet \\ & \quad (\forall s \bullet inst_typef\ f\ (mk_vartype\ s) = f\ s) \\ \wedge & \quad (\forall s\ tl \bullet inst_typef\ f\ (mk_type(s, tl)) = \\ & \quad \quad mk_type(s, map\ (inst_typef\ f)\ tl)) \end{aligned}$$

5.2 Of Terms

The problem here is to rename variables in such a way as to prevent identification of previously distinct variables and prevent separation of previously identical ones. .LP This is definitely not trivial, and I haven't yet figured out how to do it.

HOL Constant

$$ran : (* \times ** \rightarrow bool) \rightarrow (** \rightarrow bool)$$

$$ran\ f\ e = \exists x \bullet f\ (x, e)$$

HOL Constant

$$\begin{aligned} & inst_termf : \\ & \quad (string \rightarrow TYPE) \\ & \quad (* instantiations required *) \\ \rightarrow & \quad ((string \times TYPE \rightarrow bool) \\ & \quad (* free variables to avoid clashing with *) \\ & \quad \times ((string \times TYPE) \times (string \times TYPE) \rightarrow bool) \\ & \quad (* substitutions already done and still relevant *) \\ & \quad \times TERM) \\ \rightarrow & \quad ((string \times TYPE \rightarrow bool) \\ & \quad \times ((string \times TYPE) \times (string \times TYPE) \rightarrow bool) \\ & \quad \times TERM) \end{aligned}$$

$$\begin{aligned} \forall & \quad (tsubs : string \rightarrow TYPE) \\ & \quad (fvars : string \times TYPE \rightarrow bool) \\ & \quad (vsubs : (string \times TYPE) \times (string \times TYPE) \rightarrow bool) \bullet \\ (\forall s\ t \bullet & \quad inst_termf\ tsubs\ (fvars, vsubs, (mk_var\ (s,t))) \end{aligned}$$

```

=
(∃v'• vsubs ((s,t), v')) =>
  (fvars,
   vsubs,
   mk_var v')
| (inst_typef tsubs t = t) =>
  ((s,t) ∈ (ran vsubs) =>
   (let nn = s (* rename it *)
    in fvars,
        (vsubs ∩ (l_t_s [(s,t),(nn,t)])),
        mk_var(nn,t))
  | fvars ∩ (l_t_s [s,t]),
   vsubs,
   mk_var(s,t))
| let nn =
  ((s, inst_typef tsubs t) ∈ (fvars ∩ (ran vsubs))
  => s (* change to new name for variable *)
  | s)
  in (fvars,
      vsubs ∩ (l_t_s[(s,t), (nn,inst_typef tsubs t)]),
      mk_var(nn,inst_typef tsubs t)
    )
)
∧ (∀s t•
  inst_termf tsubs (fvars, vsubs, (mk_const (s,t)))
  = (fvars,
     vsubs,
     mk_const(s, inst_typef tsubs t)))
∧ (∀f a•
  inst_termf tsubs (fvars, vsubs, (mk_comb (f,a)))
  = let lr = inst_termf tsubs (fvars, vsubs, f)
    in
      let rr = inst_termf tsubs (FST lr, FST(SND lr), a)
      in
        (FST rr,
         FST(SND rr),
         mk_comb(SND(SND lr),SND(SND rr))))
∧ (∀v b•
  inst_termf tsubs (fvars, vsubs, (mk_abs (v,b)))
  = let lr = inst_termf tsubs (fvars, vsubs, v)
    in
      let rr = inst_termf tsubs (FST lr, FST(SND lr), v)
      in
        (FST rr δ ((FST lr) δ fvars),
         FST(SND rr),
         mk_abs(SND(SND lr),SND(SND rr))))

```

6 SUBSTITUTION

We will give a rather unconstructive formulation of the notion of substitution. First we need to define renaming: *rename* (*v*, *ty*) *w e* is the result of changing all instances of the variable with name *v* and type *ty* in the term *e* to have name *w*.

HOL Constant

$$\text{rename} : (\text{string} \times \text{TYPE}) \rightarrow \text{string} \rightarrow \text{FTERM} \rightarrow \text{FTERM}$$

$$\begin{aligned} & \forall \\ & (v : \text{string}) (ty : \text{TYPE}) (w : \text{string}) \\ & (vv : \text{string}) (tty : \text{TYPE}) (cc : \text{string}) \\ & (ff : \text{FTERM}) (aa : \text{FTERM}) (bb : \text{FTERM}) \\ & \bullet \\ & (\text{rename } (v, ty) w (\text{mk_fvar}(vv, tty)) = \\ & \quad ((v = vv) \wedge (ty = tty)) \Rightarrow \\ & \quad \quad \text{mk_fvar}(w, ty) \mid \text{mk_fvar}(vv, tty)) \\ & \wedge \\ & (\text{rename } (v, ty) w (\text{mk_fconst}(cc, tty)) = \\ & \quad \text{mk_fconst}(cc, tty)) \\ & \wedge \\ & (\text{rename } (v, ty) w (\text{mk_fcomb}(ff, aa)) = \\ & \quad \text{mk_fcomb}(\text{rename } (v, ty) w ff, \text{rename } (v, ty) w aa)) \\ & \wedge \\ & (\text{rename } (v, ty) w (\text{mk_fabs}(vv, tty, bb)) = \\ & \quad ((v = vv) \wedge (ty = tty)) \Rightarrow \\ & \quad \quad \text{mk_fabs}(vv, tty, bb) \mid \text{mk_fabs}(vv, tty, \text{rename } (v, ty) w bb)) \end{aligned}$$

We will need to test when two terms are α -convertible:

HOL Constant

$$\text{aconv} : \text{FTERM} \rightarrow \text{FTERM} \rightarrow \text{bool}$$

$$\begin{aligned} & \forall (t1 : \text{FTERM}) (t2 : \text{FTERM}) \bullet \\ & \text{aconv } t1 \ t2 = \\ & \quad (t1 = t2) \\ & \vee \\ & \quad (\exists t1f \ t1a \ t2f \ t2a \bullet \\ & \quad \quad (t1 = \text{mk_fcomb}(t1f, t1a)) \\ & \quad \quad \wedge \quad (t2 = \text{mk_fcomb}(t2f, t2a)) \\ & \quad \quad \wedge \quad (\text{aconv } t1f \ t2f \ \wedge \ \text{aconv } t1a \ t2a)) \\ & \vee \\ & \quad (\exists v1 \ v2 \ ty \ b1 \ b2 \bullet \\ & \quad \quad (t1 = \text{mk_fabs}(v1, ty, b1)) \\ & \quad \quad \wedge \quad (t2 = \text{mk_fabs}(v2, ty, b2)) \\ & \quad \quad \wedge \quad \text{aconv } t1 \ (\text{mk_fabs}(v1, ty, \text{rename } (v2, ty) v1 \ b2))) \end{aligned}$$

We will need to choose new variable names:

HOL Constant

$$new_var : TYPE \rightarrow (string \times TYPE \rightarrow bool) \rightarrow string$$

$$\forall ty\ vs \bullet (new_var\ ty\ vs,\ ty) \Sigma\ vs$$

Now we can define *subst*. *subst F t1* gives the term resulting from replacing every free variable in *t1* by its “image under *F*” with bound variables renamed as necessary to avoid capture. Variables which are not to be changed correspond to pairs (s, t) with $F(s, t) = mk_fvar(s, t)$.

HOL Constant

$$subst : (string \times TYPE \rightarrow FTERM) \rightarrow FTERM \rightarrow FTERM$$

$$\forall$$

$$(v : string) (ty : TYPE) (c : string)$$

$$(f : FTERM) (a : FTERM) (b : FTERM)$$

$$\bullet$$

$$(subst\ R\ (mk_fvar(v,\ ty)) = R\ (v,\ ty))$$

$$\wedge$$

$$(subst\ R\ (mk_fconst(c,\ ty)) = mk_fconst(c,\ ty))$$

$$\wedge$$

$$(subst\ R\ (mk_fcomb(f,\ a)) =$$

$$mk_fcomb(subst\ R\ f,\ subst\ R\ a))$$

$$\wedge$$

$$(subst\ R\ (mk_fabs(v,\ ty,\ b)) =$$

$$let\ RR = \in x \bullet (x = (v,\ ty)) \Rightarrow (mk_fvar\ x) \mid R\ x$$

$$in\ if(\exists w \bullet$$

$$(w \blacksquare (v,\ ty))$$

$$\wedge ((v,\ ty) \in freevars_fset\ (RR\ w))$$

$$\wedge (w \in freevars_fset\ b)$$

$$)then(\ let\ new_frees =$$

$$\ominus(image\ (freevars_fset\ o\ RR)\ (freevars_fset\ b))$$

$$in\ let\ v' = new_var\ ty\ new_frees$$

$$in\ let\ RRR = \in x \bullet (x = (v',\ ty)) \Rightarrow (mk_fvar\ x) \mid RRR\ x$$

$$in$$

$$mk_fabs(v',\ ty,\ subst\ RRR\ (rename\ (v,\ ty)\ v'\ b))$$

$$)else(\$$

$$mk_fabs(v,\ ty,\ subst\ RR\ b))$$

The inference rule **SUBST** may readily be defined in terms of the above function.

7 SEQUENTS

SML

$$new_type_abbrev('SEQ', ":(TERM) list \times TERM");;$$

8 PROOFS

SML

```

new_type 0 'FPROOF';;
new_constant ('mk_fAXp',":SEQ → FPROOF");;
new_constant ('mk_fASSUMEp',":TERM → FPROOF");;
new_constant ('mk_fREFLp',":TERM → FPROOF");;
new_constant ('mk_fBETA_CONVp',":TERM → FPROOF");;
new_constant ('mk_fSUBSTp',
  ":(FPROOF × TERM)list → TERM → FPROOF → FPROOF");;
new_constant ('mk_fABSp',":TERM → FPROOF → FPROOF");;
new_constant ('mk_fINST_TYPEp',
  ":(TYPE × TYPE)list → FPROOF → FPROOF");;
new_constant ('mk_fDISCHp',":TERM → FPROOF → FPROOF");;
new_constant ('mk_fMPp',":FPROOF → FPROOF → FPROOF");;

let FPROOF_axiom = new_axiom('FPROOF_axiom',
  ∀
    (fAXf:SEQ → *)
    (fASSUMEf:TERM → *)
    (fREFLf:TERM → *)
    (fBETA_CONVf:TERM → *)
    (fSUBSTf:(FPROOF × TERM)list → TERM → FPROOF → (* )list → * → *)
    (fABSp:TERM → FPROOF → * → *)
    (fINST_TYPEf:(TYPE × TYPE)list → FPROOF → * → *)
    (fDISCHf:TERM → FPROOF → * → *)
    (fMPf:FPROOF → FPROOF → * → * → *)•
  ∃ (fn:FPROOF → *)•
    (∀seq• fn (mk_fAXp seq)
      = fAXf s)
    ∧ (∀term• fn(mk_fASSUMEp term)
      = fASSUMEf term)
    ∧ (∀term• fn(mk_fREFLp term)
      = fREFLf term)
    ∧ (∀term• fn(mk_fBETA_CONVp term)
      = fBETA_CONVf term)
    ∧ (∀fptl term fproof•
      fn(mk_fSUBSTp fptl term fproof)
      = fSUBSTf fptl term fproof
        (map (fn o FST) fptl) (fn fproof))
    ∧ (∀term fproof•
      fn(mk_fABSp term fproof)
      = fABSp term fproof (fn fproof))
    ∧ (∀ttl fproof•
      fn(mk_fINST_TYPEp ttl fproof)
      = fINST_TYPEf ttl fproof (fn fproof))
    ∧ (∀term fproof•

```

```

      fn(mk_fDISCHp term fproof)
        = fDISCHf term fproof (fn fproof)
    ∧ (∀fproof1 fproof2 •
      fn(mk_fMPp fproof1 fproof2)
        = fMPf fproof1 fproof2 (fn fproof1) (fn fproof2))
  ");;
```

9 THEORIES

We have already defined the type environment part of a theory. A theory also has both a constant environment and a bunch of axioms:

SML

```

new_type_abbrev('TY_ENV', ":string → num → bool");;

new_type_abbrev('CON_ENV', ":string → TYPE → bool");;

new_type_abbrev('SEQS', ":SEQ → bool");;
```

z

'THEORY' "T":

```

name          :string,
parents :string → bool,
ty_env       :TY_ENV,
con_env      :CON_ENV,
extensions   :SEQS,
axioms       :SEQS,
theorems     :SEQS
```

T

SML

```

new_type_abbrev('FTHEORIES', ":THEORY list");;
```

10 THE THEORY

11 SEMANTIC DOMAINS

1. Mono-types are sets (objects of type SET).
2. A type variable assignment is a map from strings to Mono-types (SETS).
3. A type constant assignment is a map from (string X SET list) to SET.
4. Monomorphic individuals are elements of SET.
5. Polmorphic values are maps from type variable assignments to Monomorphic values.
6. A variable assignment is a map from string to polymorphic value.
7. A constant assignment is a map from string to polymorphic value.

12 ISSUES

12.1 Changes to the Language

I am strongly tempted to make some adjustments to the language. The strongest temptation I have is to outlaw overloading of variable names. Some but not all of the above specs are written on that assumption. I am also tempted to have term formation rules which are independent of the theory environment (i.e. allow constants to be used before declaration). Also to allow overloading of constants. (rules for conservative extension permitting sets of constant definitions provided the types used are pairwise non-unifiable).

12.2 Formalisation

I would like to avoid having FAILURE results on all functions which are not certain to succeed, but am not sure whether they can be avoided. The reason is to keep the specification simple. Where a function is normally used in circumstances when its arguments are known to be satisfactory, I think it is better to use a version which does not have failure codes in it. To make this possible as consistently as possible I think functions with failure results should be used only where absolutely necessary. This corresponds to programming in ML by checking with "is_?" before doing "dest_?" instead of going ahead regardless and then trapping the exception. Trapping exceptions is easier in ML but more complicated in HOL.